Learning Realtime One-Counter Automata

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1. Motivation

2. Learning deterministic finite automata

3. Learning realtime one-counter automata

4. Experimental results

```json
{
    "string": "Hello, world",
    "integer": 42,
    "double": 2.718,
    "array": ["string"],
    "object": {"anything": "correct"}
}
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Listing 1: A JSON document.

[1] For XML documents, see Chitic and Rosu, "On Validation of XML Streams Using Finite State Machines", 2004

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$\hookrightarrow$ Realtime one-counter automata and our learning algorithm!

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1. Motivation

2. Learning deterministic finite automata
   - Deterministic finite automaton
   - Active learning

3. Learning realtime one-counter automata

4. Experimental results

A deterministic finite automaton[2] (DFA) is a tuple $\mathcal{A} = (Q, \Sigma, \delta, q_0, F)$ where:

- Q is the set of states,
- Σ is the alphabet,
- $q_0 \in Q$ is the initial state,
- $F \subseteq Q$ is the set of accepting states, and
- $\delta \subseteq Q \times \Sigma \to Q$ is the transition function.

[2] Hopcroft and Ullman, *Introduction to Automata Theory, Languages and Computation*, 2000.

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The run for the word $w = a_1 \dots a_n \in \Sigma^*$ ($n \in \mathbb{N}$) is the sequence of states

$$q_0 \xrightarrow[\mathcal{A}]{a_1} p_1 \xrightarrow[\mathcal{A}]{a_2} \dots \xrightarrow[\mathcal{A}]{a_n} p_n.$$

If $p_n \in F$, the run is said accepting.

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$$q_0 \xrightarrow[\mathcal{A}]{a_1} p_1 \xrightarrow[\mathcal{A}]{a_2} \dots \xrightarrow[\mathcal{A}]{a_n} p_n.$$

If $p_n \in F$, the run is said accepting.
The language of $\mathcal{A}$ is the set

$$\mathcal{L}(\mathcal{A}) = \{w \in \Sigma^* \mid \exists q \in F, q_0 \xrightarrow[\mathcal{A}]{w} q\}.$$

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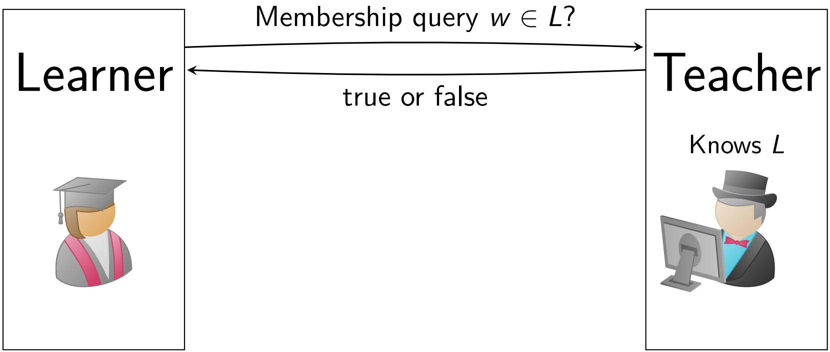
Active because the algorithm queries information during the learning process.

Figure 1: Angluin's framework Angluin, "Learning Regular Sets from Queries and Counterexamples", 1987



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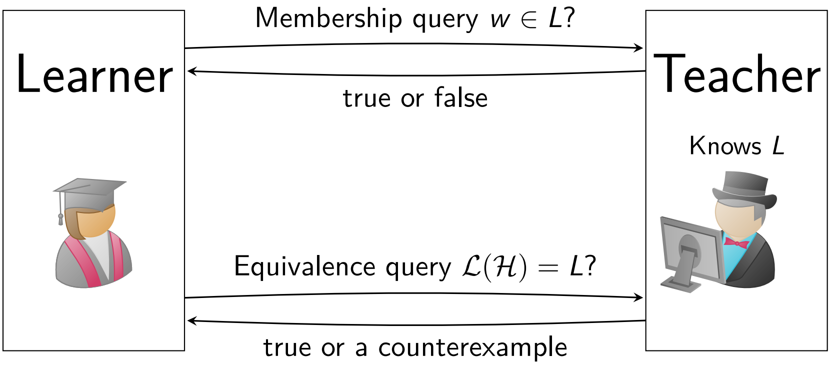

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Algorithm 1 Abstract learner for L^* [Angluin, "Learning Regular Sets from Queries and Counterexamples", 1987]

Require: The target language L
Ensure: A DFA accepting L is returned

Initialize the data structure
Fill the data structure with membership queries
while true **do**
    Make sure the data structure respects some constraints
    Construct the DFA $\mathcal{A}$
    Ask an equivalence query over $\mathcal{A}$
    if the answer is positive **then**
        return $\mathcal{A}$
    else
        Given the counterexample w, refine the data structure
        Fill the data structure with membership queries

Let $L = \{a^n b(b^* a)^m (a|b)^* \mid n, m \geq 0\}$ over $\Sigma = \{a, b\}$.

Let $u \in \Sigma^*$. For all $w \in \Sigma^*$, we look if $uw \in L$.
We construct a table where the rows are indexed by the u and the columns by the w.

Let $L = \{a^n b (b^* a)^m (a|b)^* \mid n, m \geq 0\}$ over $\Sigma = \{a, b\}$.

	ε	a	b	aa	ab	ba	bb	$\dots$
ε	0	0	1	0	1	1	1	$\dots$
a	0	0	1	0	1	1	1	$\dots$
b	1	1	1	1	1	1	1	$\dots$
aa	0	0	1	0	1	1	1	$\dots$
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$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$

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$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$

Let $u, v \in \Sigma^*$ and $L \subseteq \Sigma^*$. We say that $u \sim v$ if and only if[a]

$$\forall w \in \Sigma^*, uw \in L \Leftrightarrow vw \in L.$$

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$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$

Proposition 1

Let L be a language over Σ. Then, there is a DFA accepting L if and only if the index of $\sim$ is finite.

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The Myhill-Nerode congruence encoded in this table has a finite index. We have two equivalence classes: $[\![\varepsilon]\!]_\sim$ and $[\![b]\!]_\sim$.

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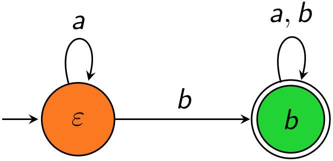

1. Motivation

2. Learning deterministic finite automata

3. Learning realtime one-counter automata
   - Realtime one-counter automata
   - Behavior graph
   - Learning algorithm

4. Experimental results

A realtime one-counter automaton (ROCA) is a tuple $\mathcal{A} = (Q, \Sigma, \delta_{=0}, \delta_{>0}, q_0, F)$ where Q, q_0, and F are defined as before, and the transition functions $\delta_{=0}$ and $\delta_{>0}$ are defined as:

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The transition relation $\xrightarrow[\mathcal{A}]{} \subseteq (Q \times \mathbb{N}) \times \Sigma \times (Q \times \mathbb{N})$ contains $(q, n) \xrightarrow[\mathcal{A}]{a} (p, m)$ if and only if

$$\begin{cases} \delta_{=0}(q, a) = (p, c) \wedge m = n + c & \text{if } n = 0 \\ \delta_{>0}(q, a) = (p, c) \wedge m = n + c & \text{if } n > 0. \end{cases}$$

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Let $w \in \Sigma^*$. The counter value of w, according to $\mathcal{A}$, is:

$$c_{\mathcal{A}}(w) = n \Leftrightarrow \exists q \in Q, (q_0, 0) \xrightarrow[\mathcal{A}]{w} (q, n).$$

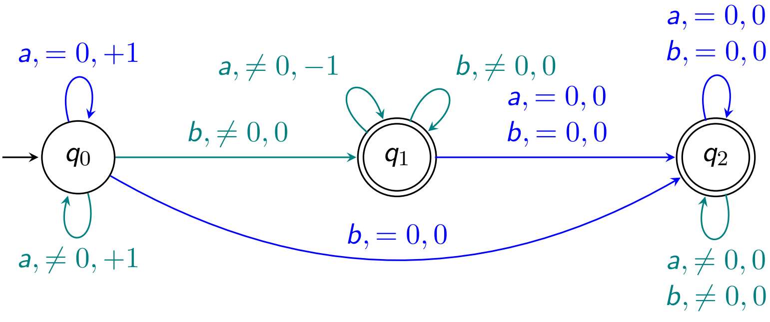
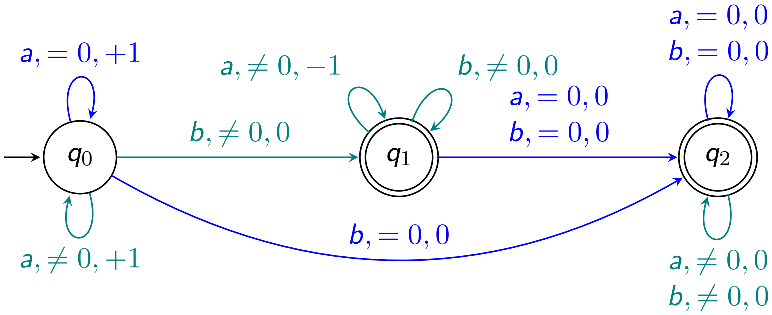


Figure 2: An ROCA $\mathcal{A}$.



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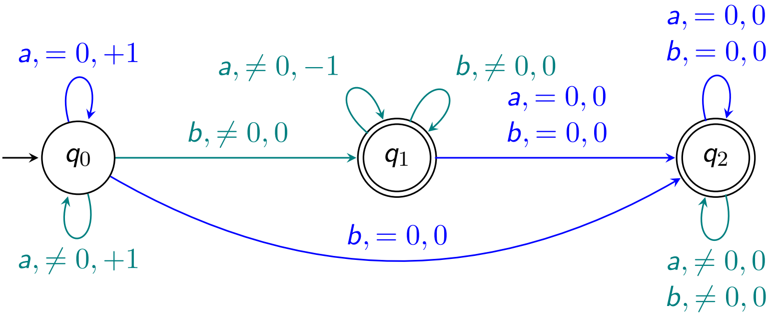
$$(q_0, 0) \xrightarrow[\mathcal{A}]{a} (q_0, 1) \xrightarrow[\mathcal{A}]{a} (q_0, 2) \xrightarrow[\mathcal{A}]{b} (q_1, 2) \xrightarrow[\mathcal{A}]{a} (q_1, 1) \xrightarrow[\mathcal{A}]{a} (q_1, 0) \xrightarrow[\mathcal{A}]{a} (q_2, 0).$$



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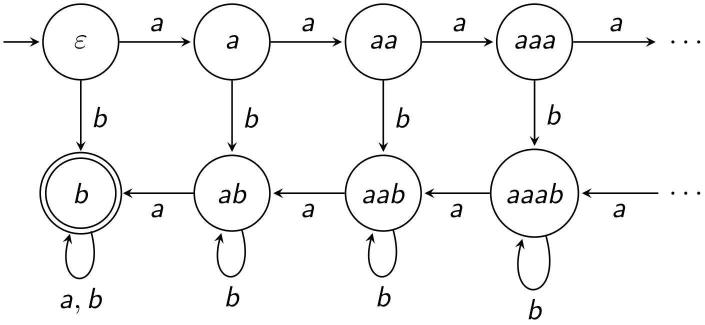
For example, let $L = \{a^n b (b^* a)^n (a|b)^* \mid n \geq 0\}$. Then, $b \equiv abba$ but $ab \not\equiv aab$.

Let $\mathcal{A}$ be an ROCA accepting L. Using the relation $\equiv$, we can construct an infinite deterministic automaton accepting L: the behavior graph of $\mathcal{A}$ $BG(\mathcal{A}) = (Q_\equiv, \Sigma, \delta_\equiv, q^0_\equiv, F_\equiv)$ with:

- $Q_\equiv = \{[\![u]\!]_\equiv \mid u \in \mathit{Pref}(L)\}$,
- $q^0_\equiv = [\![\varepsilon]\!]_\equiv$,
- $F_\equiv = \{[\![u]\!]_\equiv \mid u \in L\}$, and
- $\delta_\equiv : Q \times \Sigma \to Q$ such that $\delta([\![u]\!]_\equiv, a) = [\![ua]\!]_\equiv$ with $a \in \Sigma$ and $u, ua \in \mathit{Pref}(L)$.



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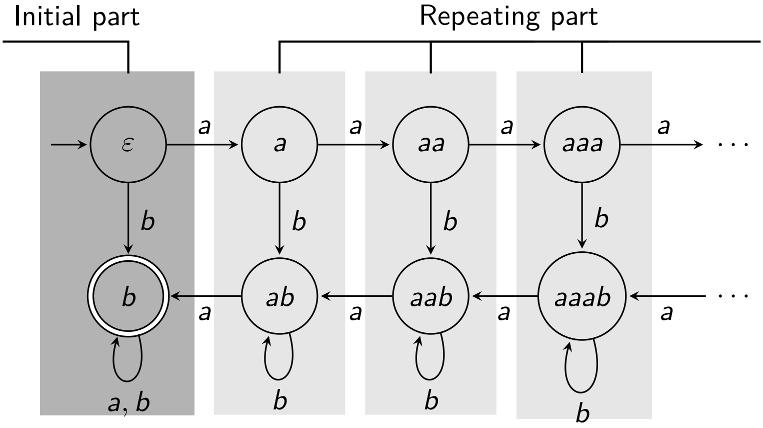

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Moreover, it is possible to construct an ROCA accepting L from $BG(\mathcal{A})$.

Let $\mathcal{A}$ be an ROCA accepting L.

[3] Based on the algorithm for VCA [Neider and Löding, *Learning visibly one-counter automata in polynomial time*, 2010].

Let $\mathcal{A}$ be an ROCA accepting L.

- Rough idea[3]: learn a sufficiently large initial fragment of $BG(\mathcal{A})$ and construct an ROCA from it.

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Let $\mathcal{A}$ be an ROCA accepting L.

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- What is an initial fragment?
  $\hookrightarrow$ $BG_\ell(\mathcal{A})$ is a subgraph of $BG(\mathcal{A})$ whose set of states is $\{[\![u]\!]_\equiv \in Q_\equiv \mid \forall x \in Pref(u), 0 \leq c_\mathcal{A}(x) \leq \ell\}$, with $\ell \in \mathbb{N}$. Let $L_\ell = \mathcal{L}(BG_\ell(\mathcal{A}))$.

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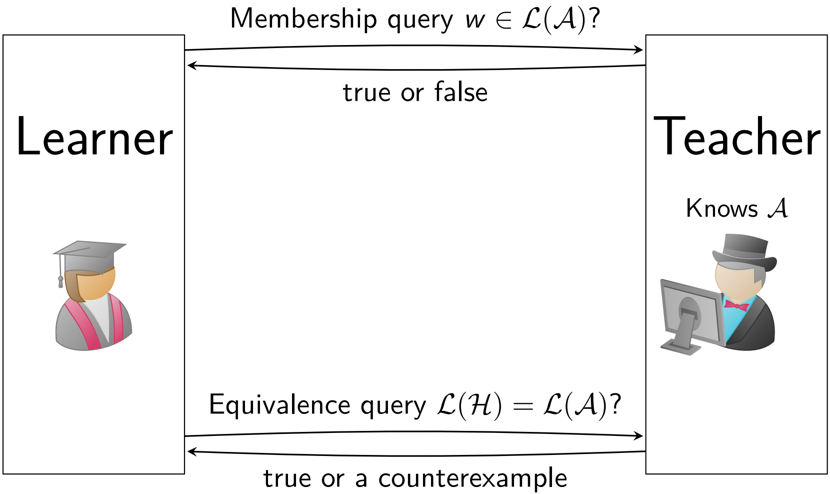
Figure 4: Adaptation of Angluin's framework for ROCAs.



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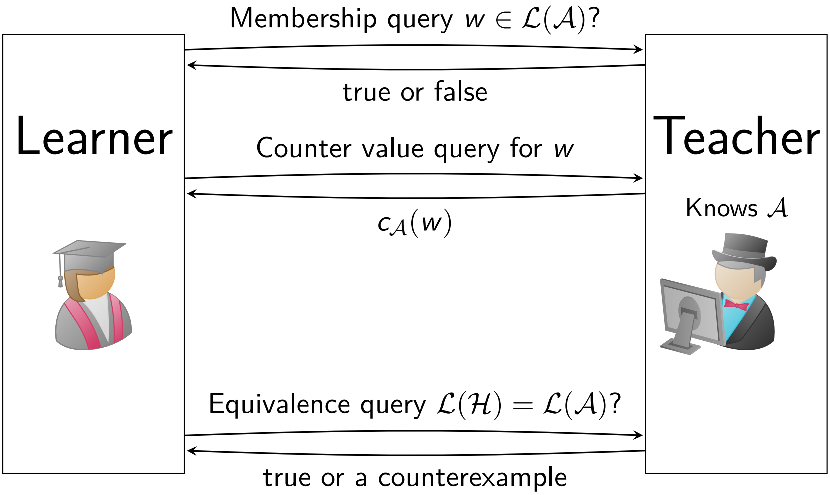

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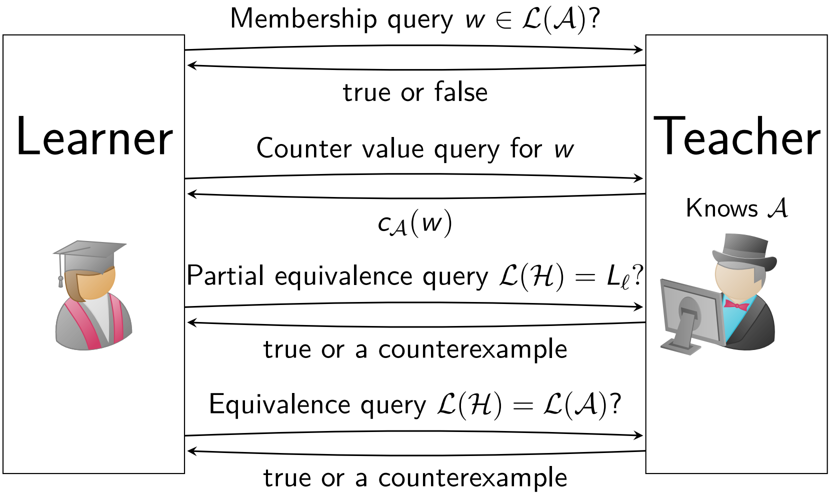

Figure 4: Adaptation of Angluin's framework for ROCAs.

Algorithm 2 Adaptation of L^* for ROCAs.

Require: A teacher knowing an ROCA $\mathcal{A}$

Ensure: An ROCA accepting the same language is returned

```
Initialize the data structure D_ℓ up to ℓ = 0
while true do
    Make D_ℓ respect the needed constraints and construct A_{D_ℓ}
    Ask a partial equivalence query over A_{D_ℓ}
    if the answer is negative then
        Update D_ℓ with the provided counterexample    ▷ ℓ is not modified
    else
        Construct all the possible ROCAs A_1, ..., A_n from A_{D_ℓ}
        Ask an equivalence query over each A_i
        if the answer is true for an A_i then return A_i
        else Select one counterexample and update D_ℓ    ▷ ℓ is increased
```

Let $\mathcal{A}$ be an ROCA accepting $L \subseteq \Sigma^*$.
An observation table up to ℓ is a tuple $\mathscr{O}_\ell = (R, S, \widehat{S}, \mathcal{L}_\ell, \mathcal{C}_\ell)$ with:

- $R \subseteq \Sigma^*$ is the prefix-closed set of representatives,
- $S \subseteq \widehat{S} \subseteq \Sigma^*$ are two suffix-closed sets of separators,
- $\mathcal{L}_\ell : (R \cup R\Sigma)\widehat{S} \to \{0, 1\}$, and
- $\mathcal{C}_\ell : (R \cup R\Sigma)S \to \{0, \dots, \ell\} \cup \{\bot\}$.

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Let $\mathit{Pref}(\mathscr{O}_\ell) = \{w \in \mathit{Pref}(us) \mid u \in R \cup R\Sigma, s \in \widehat{S}, \mathcal{L}_\ell(us) = 1\}$.
The following holds for all $u \in R \cup R\Sigma$:

- $\forall s \in \widehat{S}, \mathcal{L}_\ell(us) = 1$ if and only if $us \in L_\ell$.
- $\forall s \in S, \mathcal{C}_\ell(us) = \begin{cases} c_{\mathcal{A}}(us) & \text{if } us \in \mathit{Pref}(\mathscr{O}_\ell) \\ \bot & \text{otherwise.} \end{cases}$

	ε
ε	$0, 0$
a	$0, 1$
ab	$0, 1$
aba	$1, 0$
b	$1, 0$
aa	$0, \perp$
abb	$0, \perp$
$abaa$	$1, 0$
$abab$	$1, 0$

	ε
ε	$0, 0$
a	$0, 1$
ab	$0, 1$
aba	$1, 0$
abb	$0, 1$
b	$1, 0$
aa	$0, \bot$
$abaa$	$1, 0$
$abab$	$1, 0$
$abba$	$1, 0$
$abbb$	$0, \bot$

	ε
ε	$0, 0$
a	$0, 1$
ab	$0, 1$
aba	$1, 0$
abb	$0, 1$
$abbb$	$0, 1$
b	$1, 0$
aa	$0, \perp$
$abaa$	$1, 0$
$abab$	$1, 0$
$abba$	$1, 0$
$abbba$	$1, 0$
$abbbb$	$0, \perp$

	ε
ε	$0, 0$
a	$0, 1$
ab	$0, 1$
aba	$1, 0$
abb	$0, 1$
$abbb$	$0, 1$
b	$1, 0$
aa	$0, \bot$
$abaa$	$1, 0$
$abab$	$1, 0$
$abba$	$1, 0$
$abbba$	$1, 0$
$abbbb$	$0, \bot$

$\hookrightarrow$ Getting the algorithm to eventually finish is harder than it looks.

Theorem 3

Let $\mathcal{A}$ be an ROCA accepting a language $L \subseteq \Sigma^*$. Given a teacher for L with an automaton $\mathcal{A}$, and t the length of the longest counterexample for (partial) equivalence queries:

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Let $\mathcal{A}$ be an ROCA accepting a language $L \subseteq \Sigma^*$. Given a teacher for L with an automaton $\mathcal{A}$, and t the length of the longest counterexample for (partial) equivalence queries:

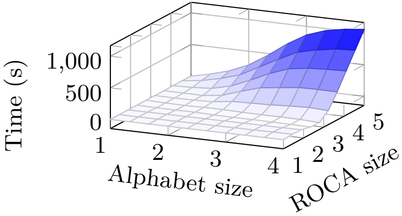
- *An ROCA accepting L can be computed in time and space exponential in $|\mathcal{A}|, |\Sigma|$ and t.*
- *The learner asks:*
  - $\mathcal{O}(t^3)$ *partial equivalence queries*
  - $\mathcal{O}(|\mathcal{A}|t^2)$ *equivalence queries*
  - *An exponential number of membership (resp. counter value) queries in $|\mathcal{A}|, |\Sigma|$, and t.*

1. Motivation

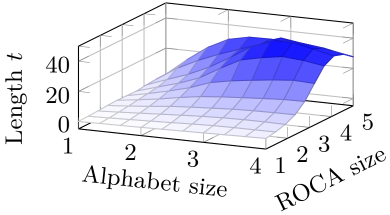
2. Learning deterministic finite automata

3. Learning realtime one-counter automata

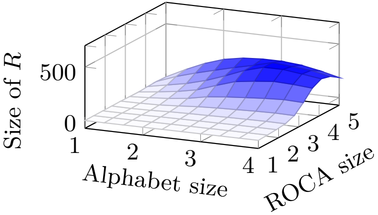
4. Experimental results



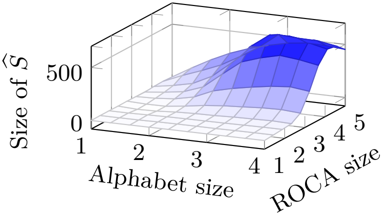
(a) Total time.



(b) Length t of the longest counterexample.



(c) Final size of R.



(d) Final size of $\widehat{S}$.

Figure 5: Experimental results for randomly generated ROCAs.

Angluin, Dana. “Learning Regular Sets from Queries and Counterexamples”. In: *Inf. Comput.* 75.2 (1987), pp. 87–106. DOI: 10.1016/0890-5401(87)90052-6. URL: https://doi.org/10.1016/0890-5401(87)90052-6.

Chitic, Cristiana and Daniela Rosu. “On Validation of XML Streams Using Finite State Machines”. In: *Proceedings of the Seventh International Workshop on the Web and Databases, WebDB 2004, June 17-18, 2004, Maison de la Chimie, Paris, France, Colocated with ACM SIGMOD/PODS 2004*. Ed. by Sihem Amer-Yahia and Luis Gravano. ACM, 2004, pp. 85–90. DOI: 10.1145/1017074.1017096. URL: https://doi.org/10.1145/1017074.1017096.

Hopcroft, John E. and Jeffrey D. Ullman. *Introduction to Automata Theory, Languages and Computation, Second Edition*. Addison-Wesley, 2000.

References II

Neider, Daniel and Christof Löding. *Learning visibly one-counter automata in polynomial time*. Tech. rep. Technical Report AIB-2010-02, RWTH Aachen (January 2010), 2010.