Verification of computer systems thanks to state machines

Gaëtan Staquet

Theoretical computer science
Computer Science Department
Science Faculty
University of Mons

Formal Techniques in Software Engineering
Computer Science Department
Science Faculty
University of Antwerp

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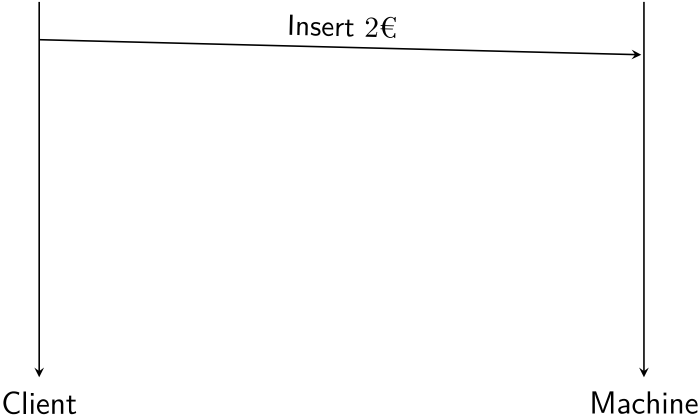
Coffee Machine – Correct execution



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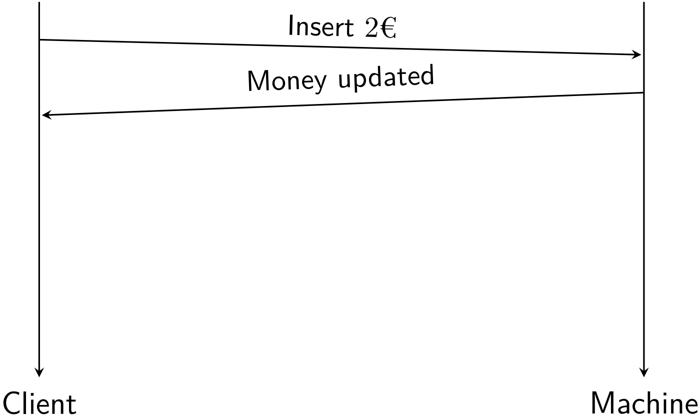

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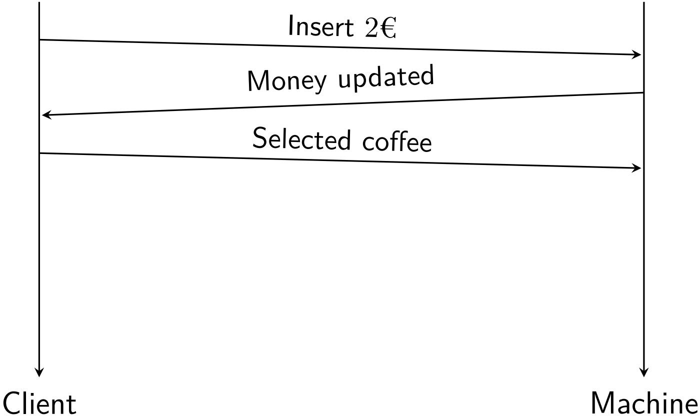

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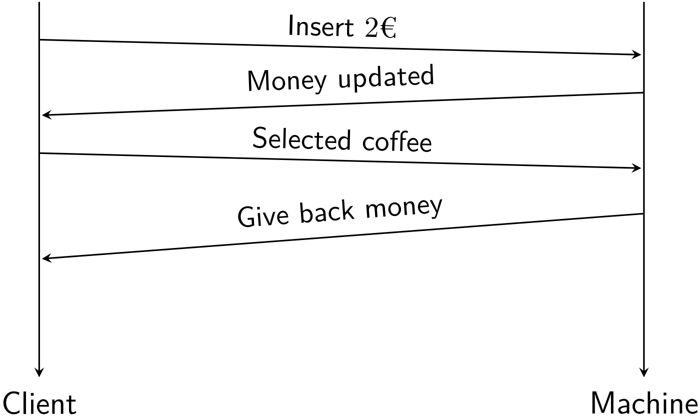

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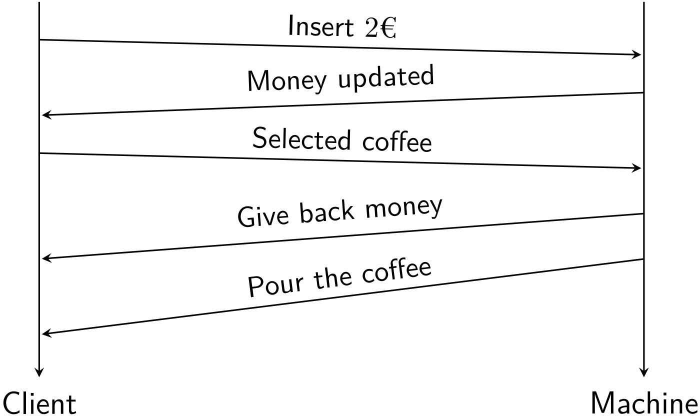

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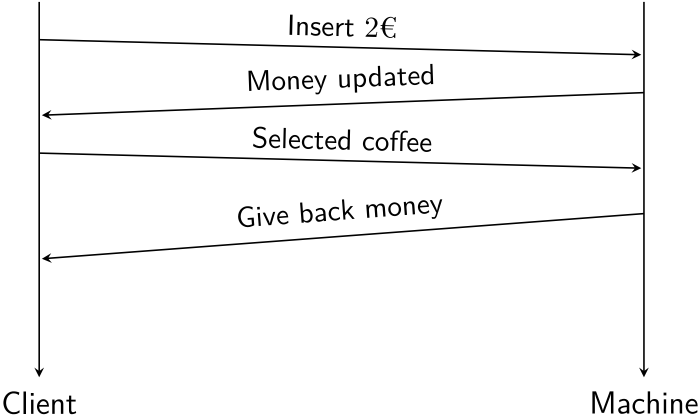

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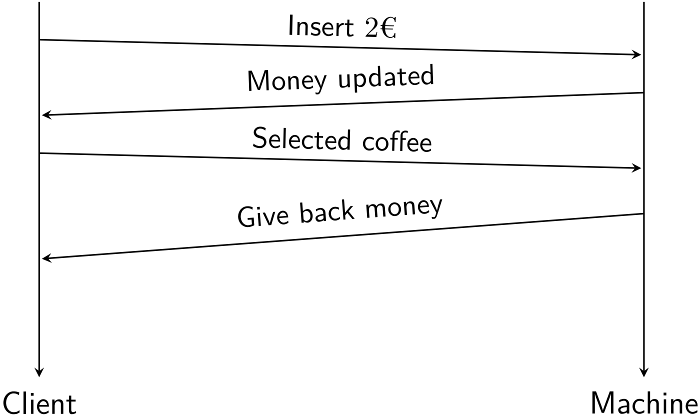

How can we detect the fault as soon as possible?

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- Construct a model $\mathcal{M}$ of the system.
- Verify if $\mathcal{M}$ satisfies the desired properties, over all possible executions.

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Here, we focus on the construction of the model.

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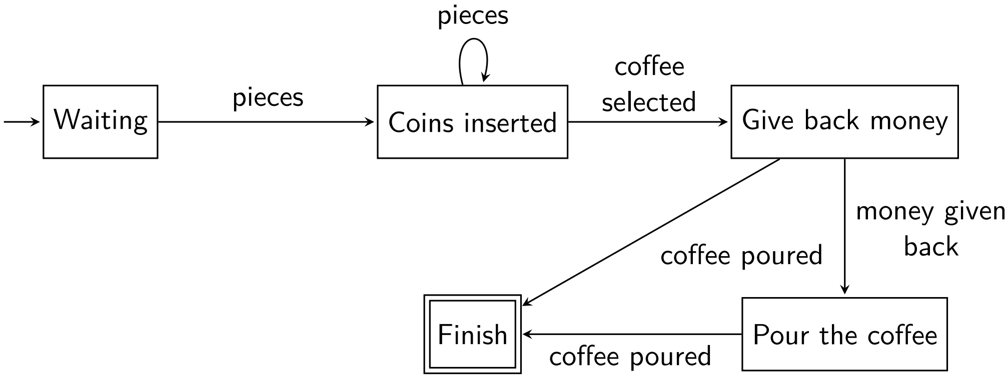

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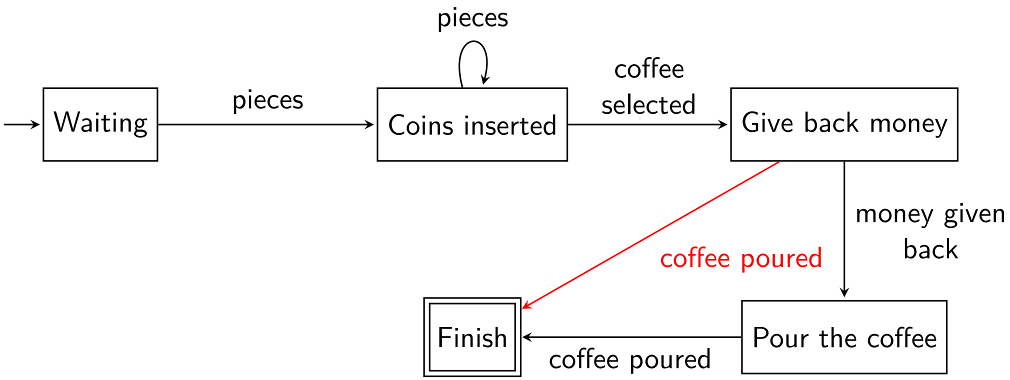

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A word $w = a_1 a_2 \dots a_n$ $(n \in \mathbb{N})$ over an alphabet Σ is a finite sequence of symbols, $a_i \in \Sigma$. The empty word is denoted by ε.

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A language L over an alphabet Σ is a set of words.

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$\Sigma = \{a, b\}$ is an alphabet.
$w = ababb$ is a word over Σ.
$L' = \{\varepsilon, a, b\}$ and $L = \{w \mid w$ has an even number of a and an odd number of $b\}$ are two languages over Σ.

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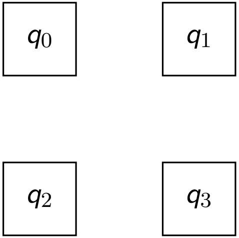

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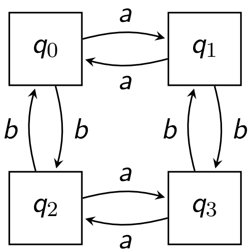

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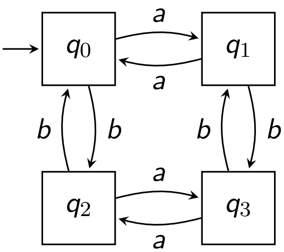

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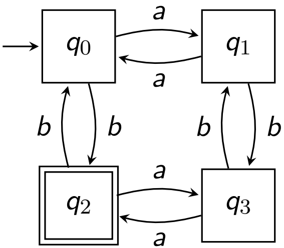
- $q_0 \in Q$ the initial state;
- $F \subseteq Q$ the set of final states.



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Which model?

Let $w = a_1 a_2 \ldots, a_n \in \Sigma^*$. The run of $\mathcal{A}$ over w is the sequence of states

$$p_1 \xrightarrow{a_1} p_2 \xrightarrow{a_2} p_3 \xrightarrow{a_3} \ldots \xrightarrow{a_n} p_{n+1}$$

such that $p_1 = q_0$ and $\forall i, \delta(p_i, a_i) = p_{i+1}$.

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Let $w = ababb$. The corresponding run is

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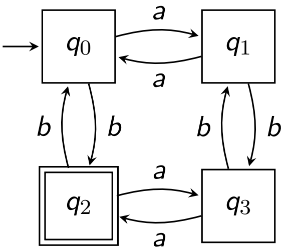

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If $p_{n+1} \in F$, then w is accepted by $\mathcal{A}$.

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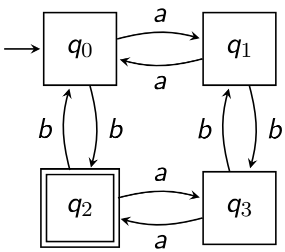

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Which model?

The language of $\mathcal{A}$ is the set of all accepted words, i.e.,

$$\mathcal{L}(\mathcal{A}) = \{w \mid \exists p \in F, q_0 \xrightarrow{w} p\}.$$

Example 3

The language of $\mathcal{A}$ is

$$\mathcal{L}(\mathcal{A}) = \{w \mid w \text{ has an even number of } a \text{ and an odd number of } b\}.$$

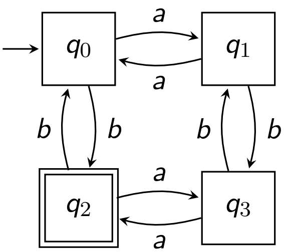

Figure 1: A DFA $\mathcal{A}$.

Infinite table

Let $L = \{w \mid w \text{ has an even number of } a \text{ and an odd number of } b\}$.

Let $u \in \Sigma^*$. For all $w \in \Sigma^*$, we check whether $uw \in L$.
We construct a table where the rows are the u and the columns the w.

Infinite table

Let $L = \{w \mid w \text{ has an even number of } a \text{ and an odd number of } b\}$.

	ε	a	b	aa	ab	ba	bb	$\dots$
ε	0	0	1	0	0	0	0	$\dots$
a	0	0	0	0	1	1	0	$\dots$
b	1	0	0	1	0	0	1	$\dots$
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The table contains in fact four different rows.

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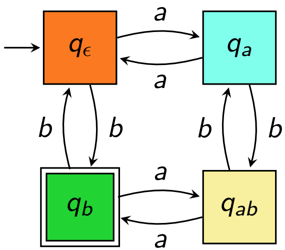

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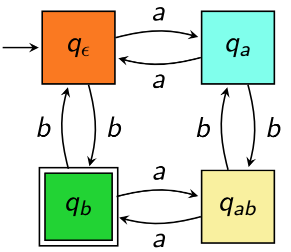

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$\hookrightarrow$ A finite table is enough.

How to learn a table?

Figure 2: Angluin's framework.[1]

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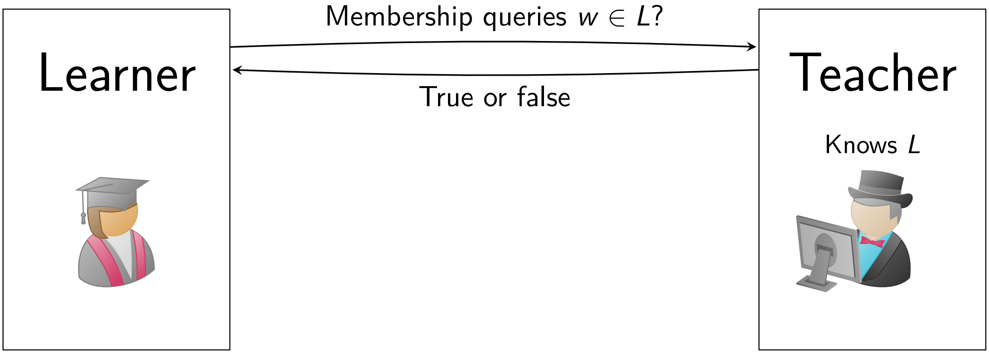

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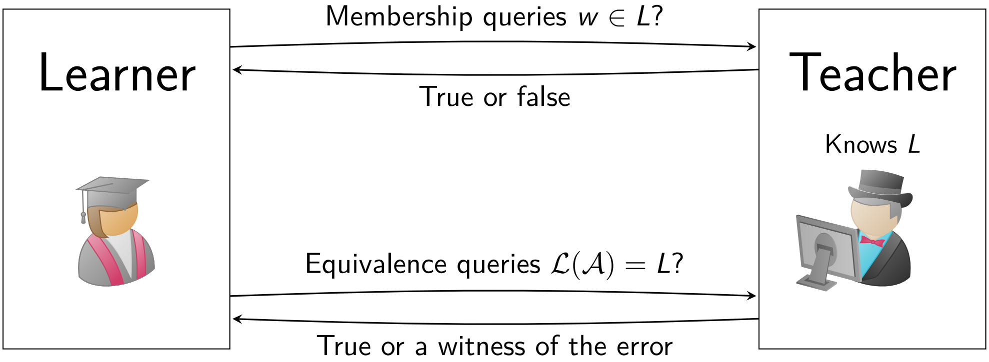

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$\hookrightarrow$ It depends on the exact problem.

JSON Documents

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  },
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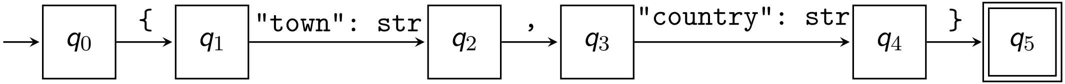

Figure 3: An automaton for the value of "place".

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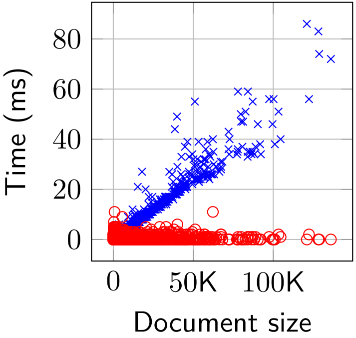

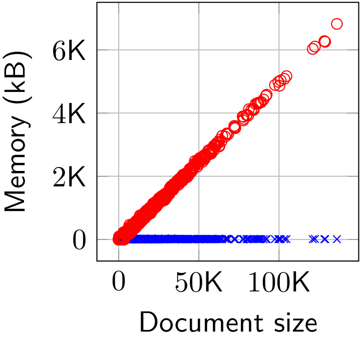

Figure 4: Experimental results for our JSON documents validation algorithm. Blue crosses given the values for our algorithm, and the red circles for the “classical” algorithm.

References I

Angluin, Dana. "Learning Regular Sets from Queries and Counterexamples". In: *Inf. Comput.* 75.2 (1987), pp. 87–106. DOI: 10.1016/0890-5401(87)90052-6. URL: https://doi.org/10.1016/0890-5401(87)90052-6.

Bruyère, V., G. A. Pérez, and G. Staquet. *Validating JSON Documents with Learned VPAs*. Pre-print. Soumis à TACAS 2023. F.R.S.-FNRS, Universités de Mons et d'Anvers, 2022.