Learning Realtime One-Counter Automata

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1. Motivation: model checking
2. Learning a DFA
3. Realtime one-counter automata
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5. Implementation and future work

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$\hookrightarrow$ The family of deterministic finite automata (DFAs) which can be learned by an active learning algorithm, such as L^*.[1]

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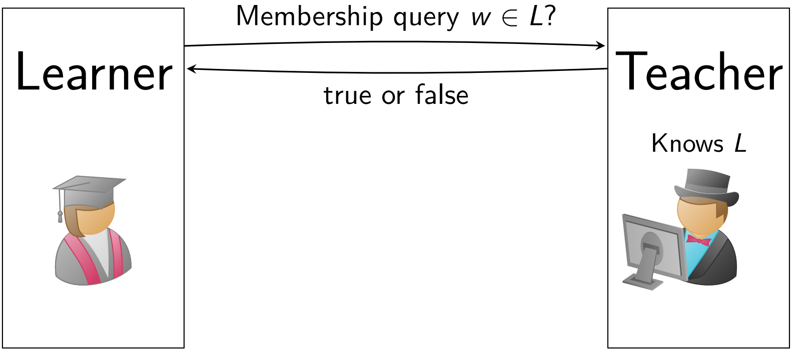
Figure 1: Angluin's framework [Angluin, "Learning Regular Sets from Queries and Counterexamples", 1987].



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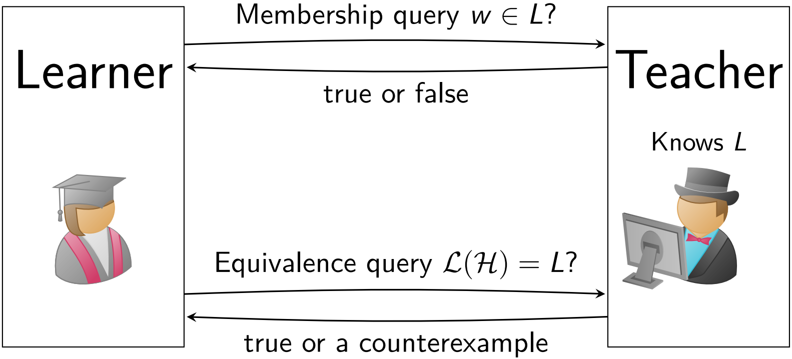

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$\hookrightarrow$ We add a natural counter.

A realtime one-counter automaton (ROCA) is a tuple $\mathcal{A} = (Q, \Sigma, \delta_{=0}, \delta_{>0}, q_0, F)$ with:

- Q is the set of states,
- Σ is the alphabet,
- $\delta_{=0}$ and $\delta_{>0}$ are the transition functions:

$$\delta_{=0} : Q \times \Sigma \to Q \times \{0, +1\}$$
$$\delta_{>0} : Q \times \Sigma \to Q \times \{-1, 0, +1\}$$

- q_0 is the initial state, and
- $F \subseteq Q$ is the set of accepting states.

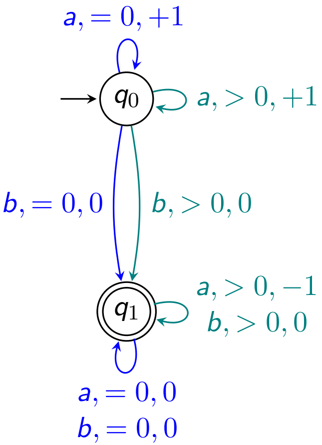
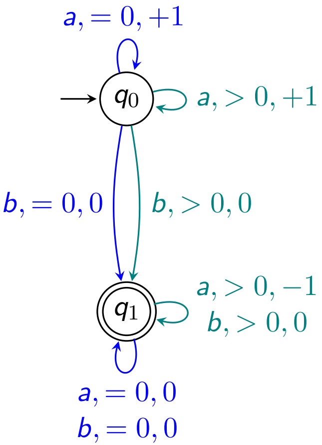


Figure 2: A realtime one-counter automaton.

An ROCA defines a configuration graph where states are $Q \times \mathbb{N}$.



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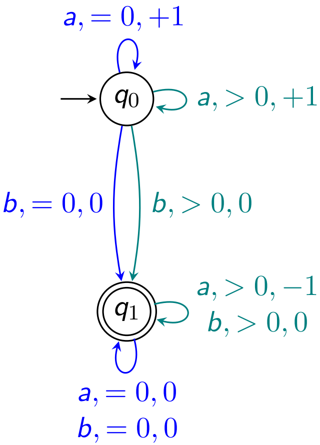
An ROCA defines a configuration graph where states are $Q \times \mathbb{N}$.
We have the following run for *aba*:

$$(q_0, 0) \xrightarrow[\mathcal{A}]{a} (q_0, 1) \xrightarrow[\mathcal{A}]{b} (q_1, 1) \xrightarrow[\mathcal{A}]{a} (q_1, 0)$$



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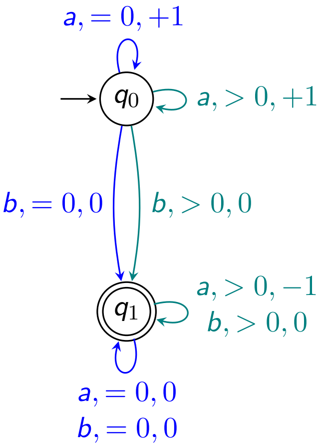

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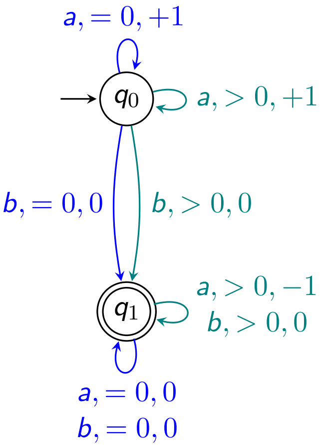
Since $q_1 \in F$ and $c_{\mathcal{A}}(aba) = 0$, *aba* is accepted by $\mathcal{A}$.



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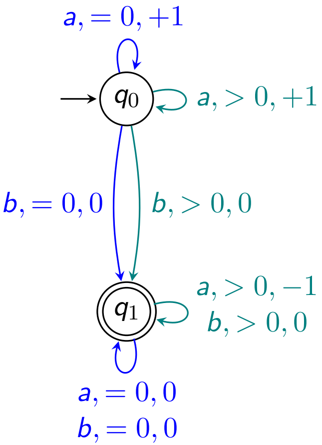
Since $q_1 \in F$ and $c_{\mathcal{A}}(aba) = 0$, *aba* is accepted by $\mathcal{A}$.
We can show that the language of $\mathcal{A}$ is

$$\mathcal{L}(\mathcal{A}) = \{a^n b (b^* a)^n \{a, b\}^* \mid n \in \mathbb{N}\}.$$



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Theorem 1

Let L be the language of some ROCA $\mathcal{A}$. It is possible to learn an ROCA accepting L in an exponential time and space complexities in $|Q|$ and $|\Sigma|$.

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For DFAs, we learn an equivalence relation called the Myhill-Nerode congruence, from which we can construct the minimal DFA accepting the target language.

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Let $\mathcal{A}$ be an ROCA accepting L, and $u, v \in \Sigma^*$. We say that $u \equiv v$ if and only if $\forall w \in \Sigma^*$, we have[2]

$$uw \in L \Leftrightarrow vw \in L,$$
$$uw, vw \in Pref(L) \Rightarrow c_{\mathcal{A}}(uw) = c_{\mathcal{A}}(vw).$$

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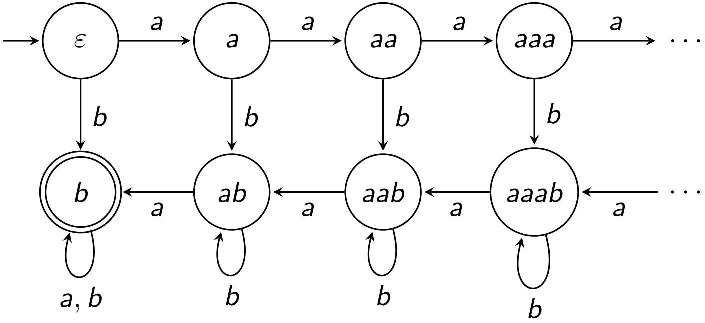
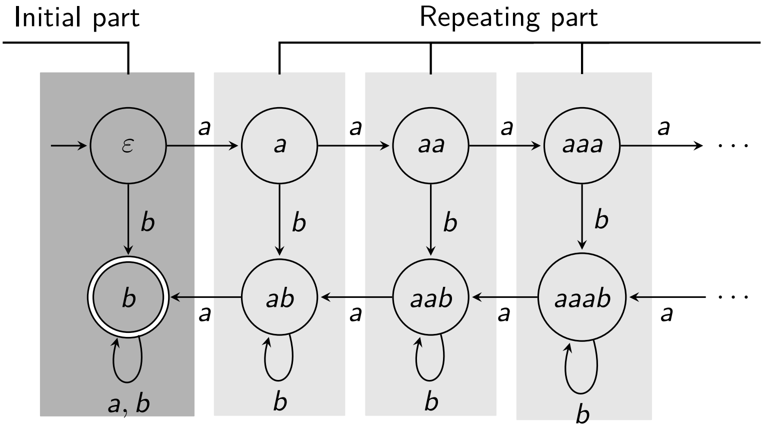


Figure 3: A behavior graph constructed from $\equiv$.



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Let $\mathcal{A}$ be an ROCA and $BG(\mathcal{A})$ be its behavior graph. Then, $BG(\mathcal{A})$ has an ultimately periodic structure.

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We fix a counter limit ℓ and we learn the minimal DFA that accepts L up to ℓ, denoted by L_ℓ.[3]

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Moreover, if ℓ is large enough, it is possible to construct an ROCA accepting L from $\mathcal{H}$.

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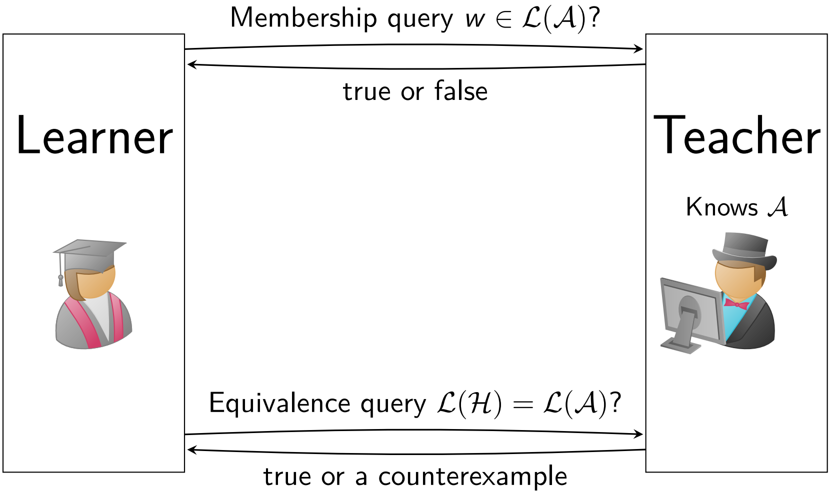
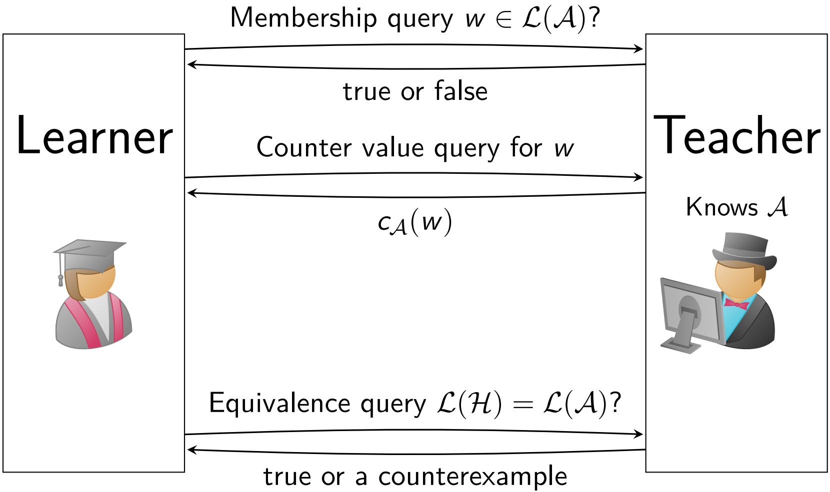


Figure 4: Adaptation of Angluin's framework for ROCAs.



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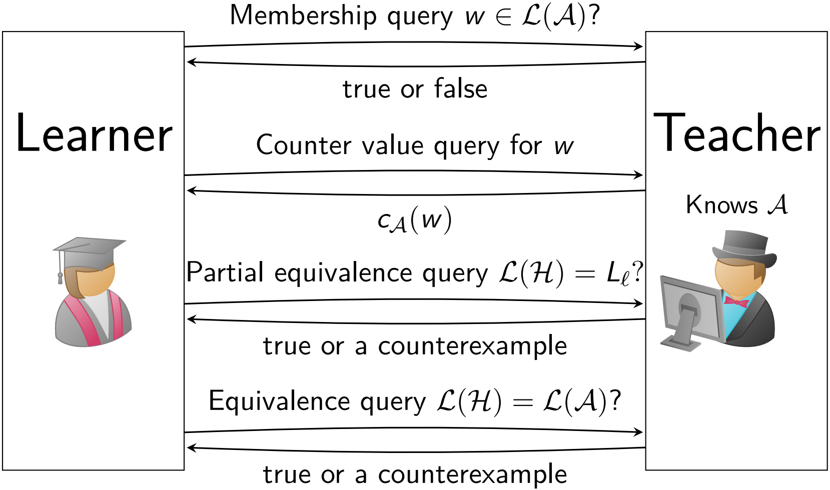

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Theorem 4

Let $\mathcal{A}$ be an ROCA accepting a language $L \subseteq \Sigma^*$. Given a teacher for L, which answers membership, counter value, and (partial) equivalence queries, an ROCA accepting L can be computed in time and space exponential in $|Q|, |\Sigma|$ and t, where t is the length of the longest counterexample returned by the teacher on (partial) equivalence queries.

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- *A number of membership (resp. counter value) queries which is exponential in $|Q|, |\Sigma|$ and t.*

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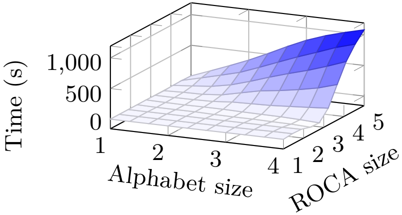
  - For instance, the algorithm never stops if we have a single set of separators.

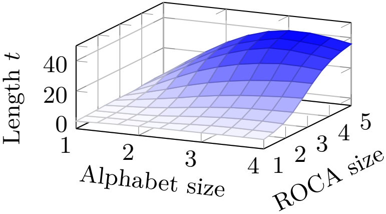
We implemented our algorithm in Java using AutomataLib and LearnLib.

We evaluated the performance on two types of benchmarks:

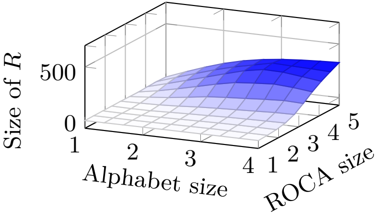
1. On randomly generated ROCAs.
2. On JSON documents.



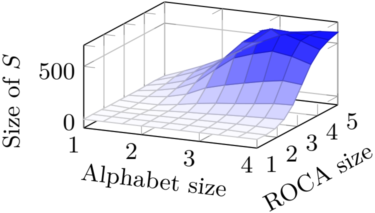
(a) Total time.



(b) Length t of the longest counterexample.



(c) Final size of R.



(d) Final size of $\widehat{S}$.

Figure 5: Experimental results for randomly generated ROCAs.

For the JSON based benchmarks, the teacher has a JSON schema which details how a document should be structured.

```
{
    "type": "object",
    "properties": {
        "subList": {
            "type": "array",
            "items": {"$ref": "#"}
        }
    }
}
```

Listing 1: A JSON schema.

Schema	TO (1h)	Time (s)	t	$\lvert R \rvert$	$\lvert \widehat{S} \rvert$	$\lvert \mathcal{A} \rvert$	$\lvert \Sigma \rvert$
1	0	16.39	31.00	55.55	32.00	33.00	19.00
2	27	1045.64	12.99	57.84	33.74	44.29	14.70
3	19	922.19	49.49	171.94	50.49	51.16	9.00

Table 1: Results for JSON documents.

For future work:

- Remove partial equivalence queries by working with more recent learning algorithms, such as TTT by Isberner et al.[4] or $L^{\#}$ by Vaandrager et al.[5]
- Lowering the complexity.

Currently, we are working on extending the use-case on JSON documents to be usable in practice.

[4] Isberner, Howar, and Steffen, "The TTT Algorithm: A Redundancy-Free Approach to Active Automata Learning", 2014.

[5] Vaandrager et al., "A New Approach for Active Automata Learning Based on Apartness", 2021.

- Angluin, Dana. "Learning Regular Sets from Queries and Counterexamples". In: *Inf. Comput.* 75.2 (1987), pp. 87–106. DOI: 10.1016/0890-5401(87)90052-6. URL: https://doi.org/10.1016/0890-5401(87)90052-6.
- Isberner, Malte, Falk Howar, and Bernhard Steffen. "The TTT Algorithm: A Redundancy-Free Approach to Active Automata Learning". In: *Runtime Verification - 5th International Conference, RV 2014, Toronto, ON, Canada, September 22-25, 2014. Proceedings*. Ed. by Borzoo Bonakdarpour and Scott A. Smolka. Vol. 8734. Lecture Notes in Computer Science. Springer, 2014, pp. 307–322. DOI: 10.1007/978-3-319-11164-3_26. URL: https://doi.org/10.1007/978-3-319-11164-3%5C_26.
- Neider, Daniel and Christof Löding. *Learning visibly one-counter automata in polynomial time*. Tech. rep. Technical Report AIB-2010-02, RWTH Aachen (January 2010), 2010.

Vaandrager, Frits W. et al. “A New Approach for Active Automata Learning Based on Apartness”. In: *CoRR* abs/2107.05419 (2021). arXiv: 2107.05419. URL: https://arxiv.org/abs/2107.05419.