Automata with Timers

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Many computer systems have timing constraints:

- Network protocols;
- Schedulers;
- Embedded systems;
- In general, real-time systems.

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BUT timed automata are hard to construct and understand.

We focus on properties that can be represented with timers: automata with timers.

Timed automata

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Automata with timers

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- Clocks go from 0 to infinity;
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Automata with timers

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▶ We know the current value of the clocks;	▶ We do not know the current value of the timers;
▶ Timed automata are more expressive;	▶ Automata with timers are more restrictive;
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- Well-known model.

Automata with timers

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- We do not know the current value of the timers;
- Automata with timers are more restrictive;
- Future work: learning algorithm;
- This work studies some properties of automata with timers.

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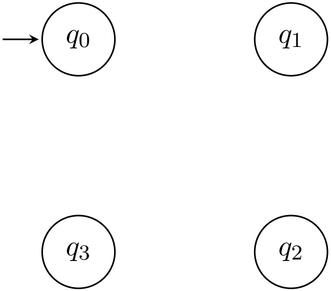

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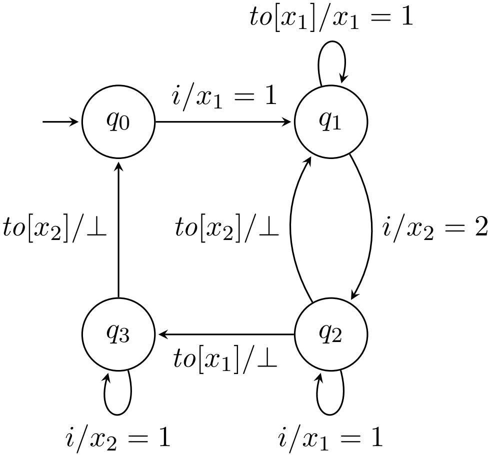
- δ is the transition function.



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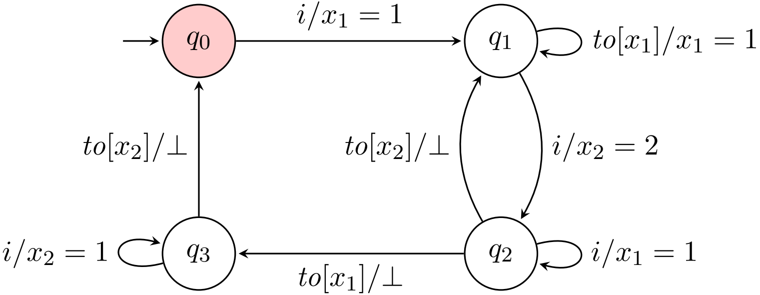
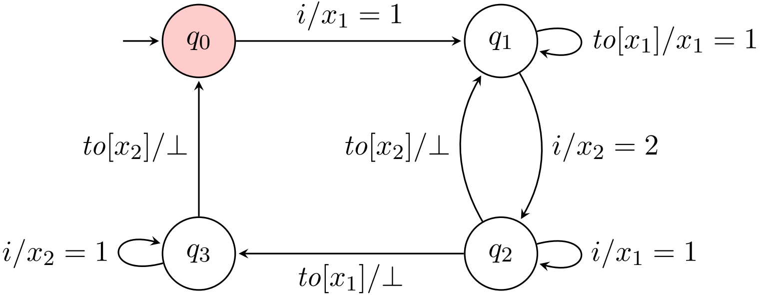


Figure 2: The same AT.

$(q_0, \emptyset)$



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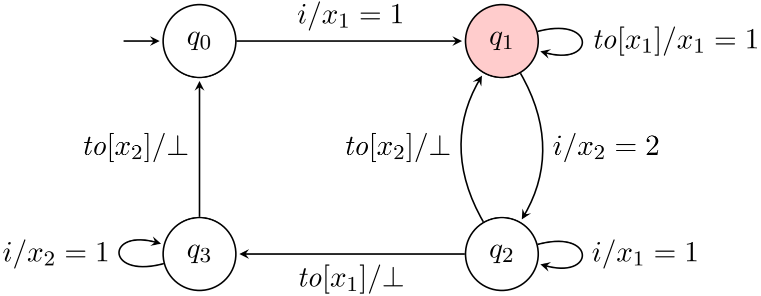

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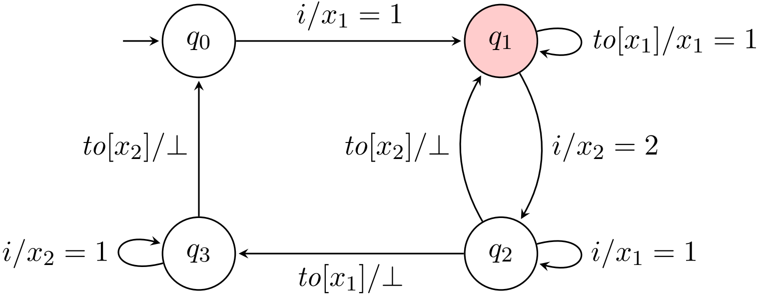

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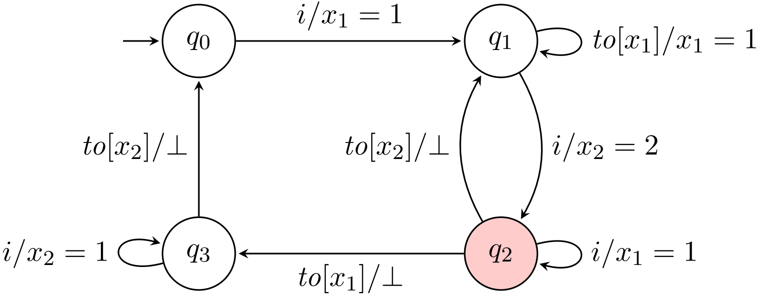

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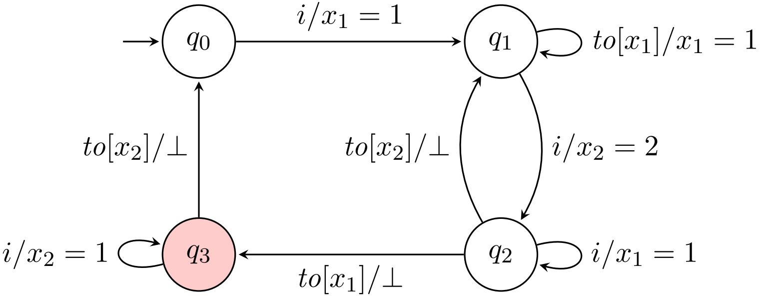

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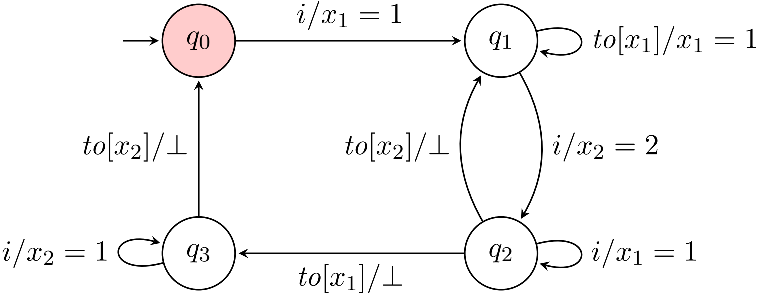

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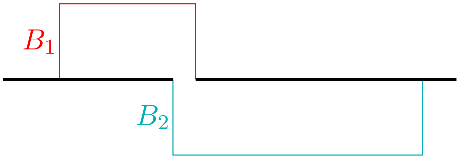

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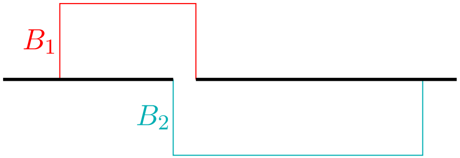

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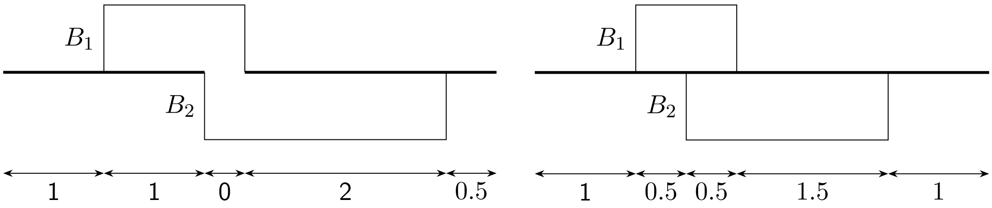
We have concurrent actions.

We can avoid this concurrency and still see the same sequence of actions.



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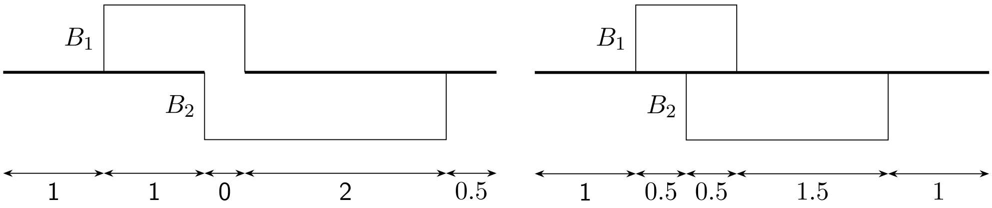

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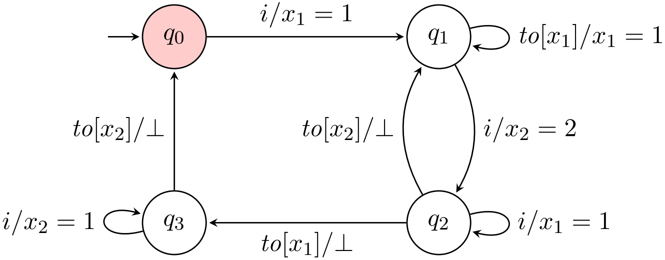
Is it always possible?



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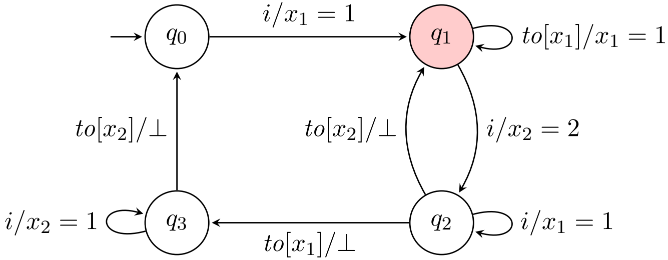

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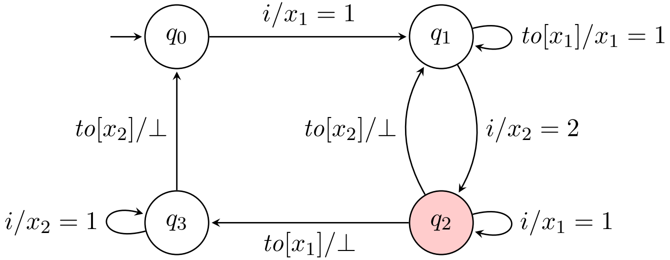

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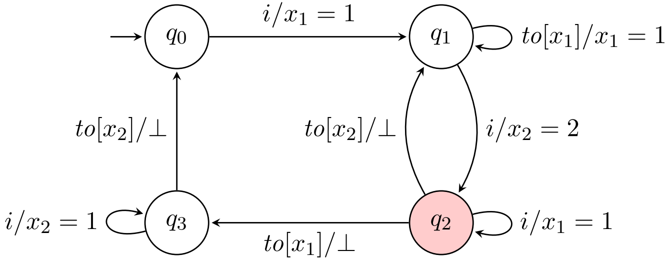

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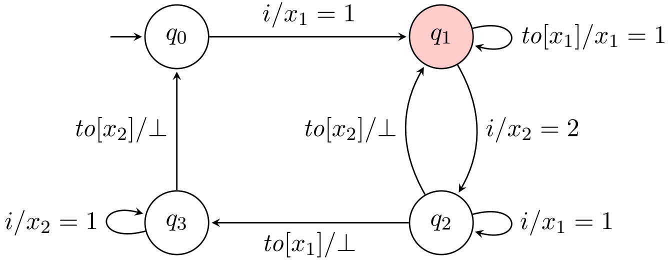

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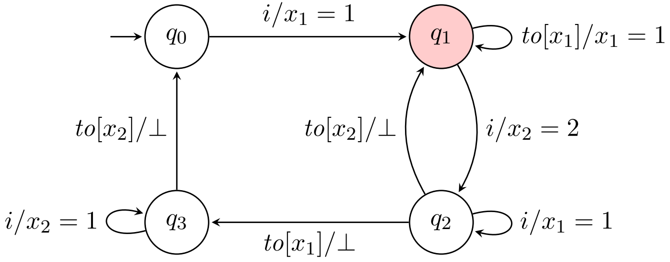

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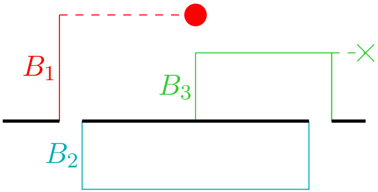

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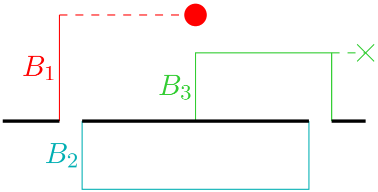

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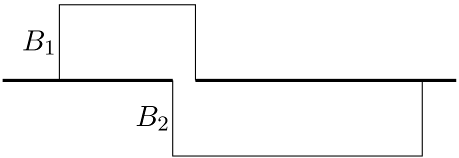
We cannot avoid this concurrency and still see the same sequence of actions.



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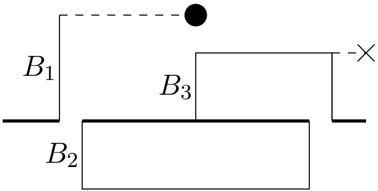
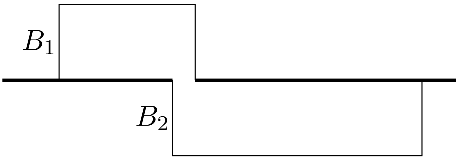


(b) Cannot be avoided.

Figure 7: Some concurrency can be avoided, some not.



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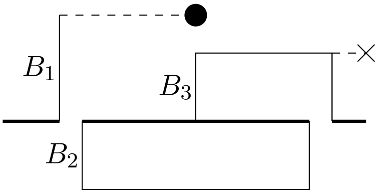

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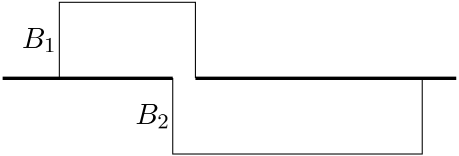
Can we characterize when it is possible to remove the concurrency?



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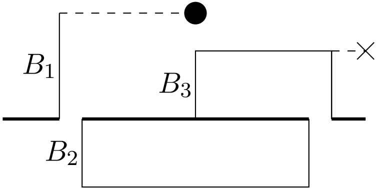

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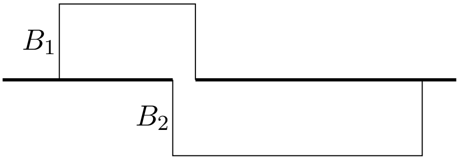
Theorem 2 (Contribution)

Deciding whether an AT contains an execution in which some concurrency cannot be avoided is PSPACE*-hard and in* 3EXP*.*



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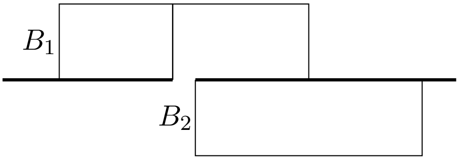

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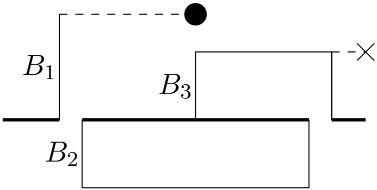
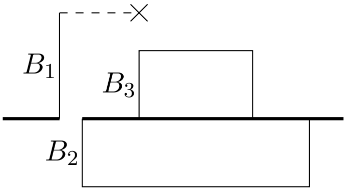
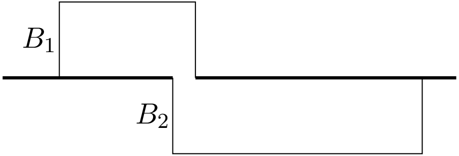


(c) Cannot be wiggled.

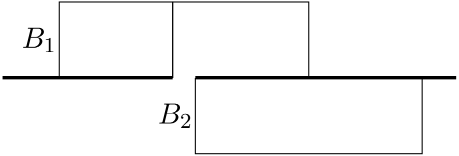


(d) Can be wiggled.

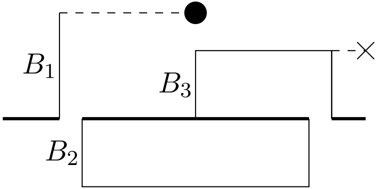
Figure 8: Not all runs can be wiggled.



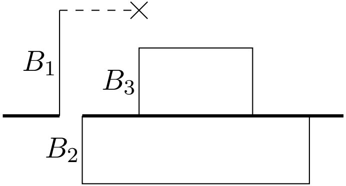
(a) $B_2 \prec B_1$.



(b) $B_1 \prec B_2$.

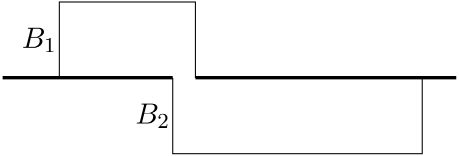


(c) $B_1 \prec B_2, B_3 \prec B_1, B_2 \prec B_3$.



(d) $B_1 \prec B_2$.

Figure 9: Define an order $\prec$ over the blocks, based on races.



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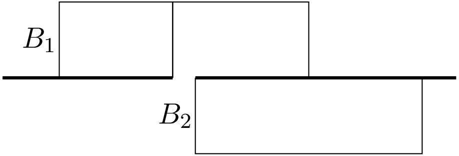
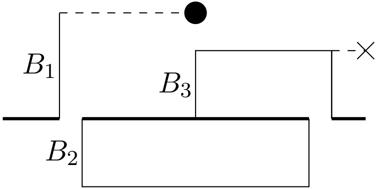
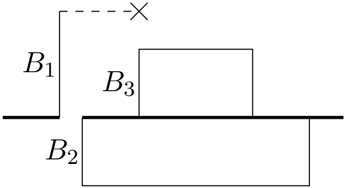


(b) $B_1 \prec B_2$.



(c) $B_1 \prec B_2, B_3 \prec B_1, B_2 \prec B_3$.



(d) $B_1 \prec B_2$.

Figure 9: Define an order $\prec$ over the blocks, based on races.

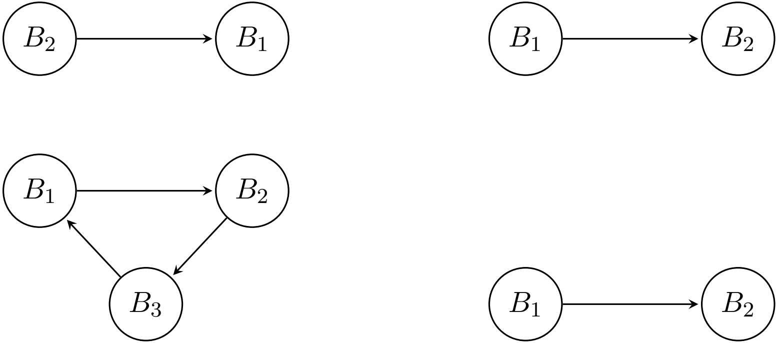


Figure 10: Block graphs defined from the blocks and $\prec$.

Proposition 3 (Contribution)

A timed run ρ can be wiggled if and only if its block graph is acyclic.

$\Rightarrow$ By contraposition, we have a cycle.
If a block has...

- A predecessor? It cannot move left.
- A successor? It cannot move right.
- Both? It cannot move at all.

Thus, ρ cannot be wiggled since we have a cycle.

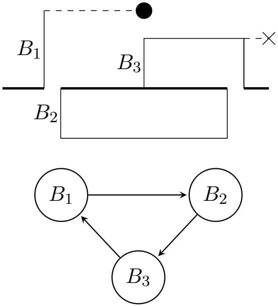


Figure 11: We have a cycle.

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A timed run ρ can be wiggled if and only if its block graph is acyclic.

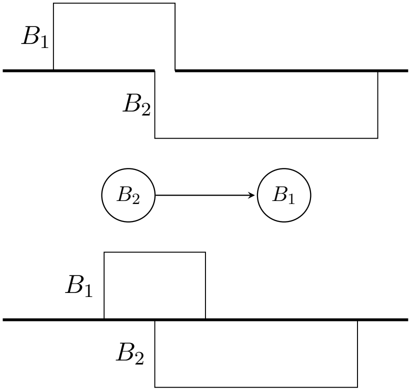

Figure 12: We change delays.

$\Leftarrow$ The graph is acyclic. Compute its topological sort and move the “last” block to the right.
$\hookrightarrow$ obtain ρ' with the same sequence of actions as ρ but ρ' contains strictly less races.
Repeat until all races are removed.

Theorem 4 (Contribution)

An AT contains a run that cannot be wiggled if and only if the block graph of that run is cyclic.

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- i.e., there are concurrent actions

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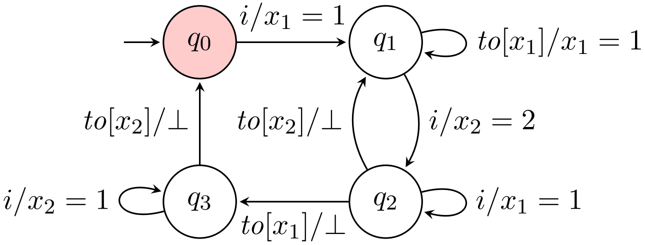
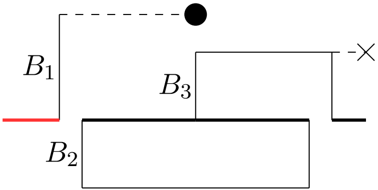
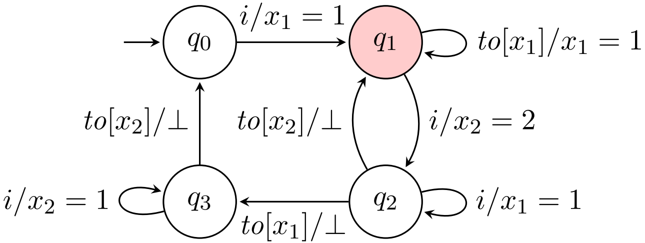
- i.e., there are concurrent actions inducing a cyclic block graph.

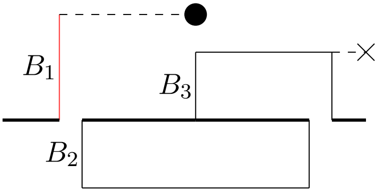
Let us illustrate using our run with unavoidable concurrencies.



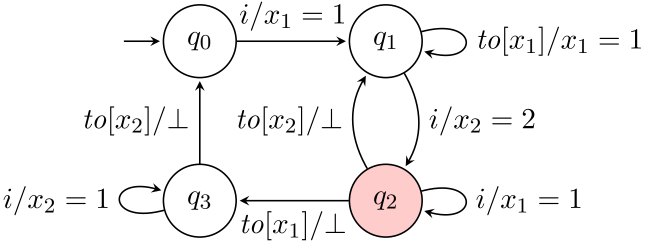


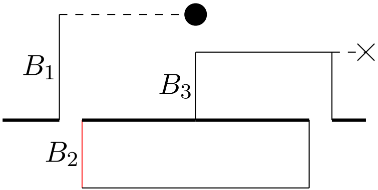
$$(q_0, \emptyset, \emptyset) \xrightarrow{\tau} (q_0, \emptyset, \emptyset)$$



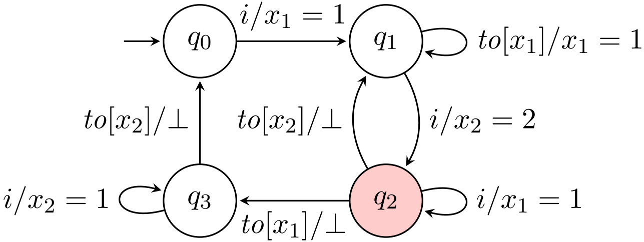


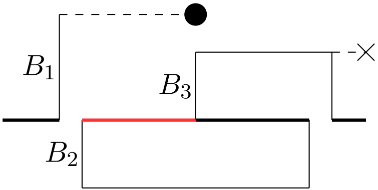
$$(q_0, \emptyset, \emptyset) \xrightarrow{\tau} (q_0, \emptyset, \emptyset) \xrightarrow{(i, x_1)} (q_1, x_1 = 1, \emptyset)$$



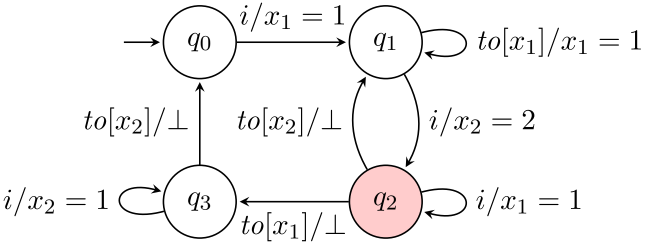


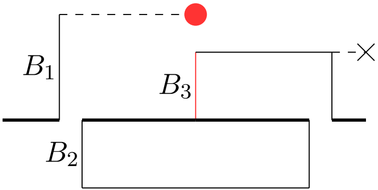
$$(q_0, \emptyset, \emptyset) \xrightarrow{\tau} (q_0, \emptyset, \emptyset) \xrightarrow{(i,x_1)} (q_1, x_1 = 1, \emptyset) \xrightarrow{(i,x_2)} (q_2, x_1 = 1 \wedge x_2 = 2, \emptyset)$$





$$(q_0, \emptyset, \emptyset) \xrightarrow{\tau} (q_0, \emptyset, \emptyset) \xrightarrow{(i,x_1)} (q_1, x_1 = 1, \emptyset) \xrightarrow{(i,x_2)} (q_2, x_1 = 1 \wedge x_2 = 2, \emptyset)$$
$$\xrightarrow{\tau} (q_2, 0 < x_1 < 1 \wedge x_2 - x_1 = 1, \emptyset) \xrightarrow{\tau} (q_2, x_1 = 0 \wedge x_2 = 1, \emptyset)$$





$$(q_0, \emptyset, \emptyset) \xrightarrow{\tau} (q_0, \emptyset, \emptyset) \xrightarrow{(i, x_1)} (q_1, x_1 = 1, \emptyset) \xrightarrow{(i, x_2)} (q_2, x_1 = 1 \wedge x_2 = 2, \emptyset)$$

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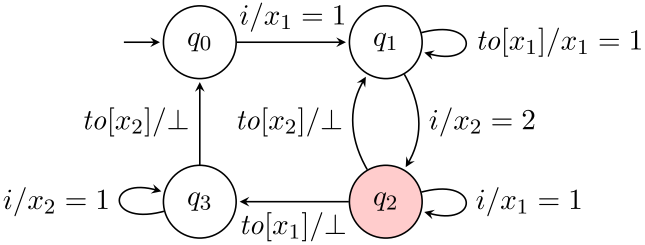
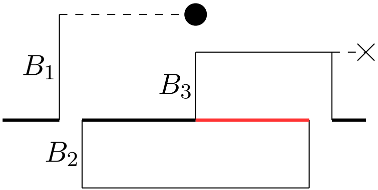
$$\xrightarrow{(i, x_1)} (q_2, x_1 = 1 = x_2, \{x_1\}) \xrightarrow{\mathrm{di}[x_1]} (q_2, x_1 = 1 = x_2, \emptyset)$$





$$
\begin{aligned}
(q_0, \emptyset, \emptyset) &\xrightarrow{\tau} (q_0, \emptyset, \emptyset) \xrightarrow{(i,x_1)} (q_1, x_1 = 1, \emptyset) \xrightarrow{(i,x_2)} (q_2, x_1 = 1 \wedge x_2 = 2, \emptyset) \\
&\xrightarrow{\tau} (q_2, 0 < x_1 < 1 \wedge x_2 - x_1 = 1, \emptyset) \xrightarrow{\tau} (q_2, x_1 = 0 \wedge x_2 = 1, \emptyset) \\
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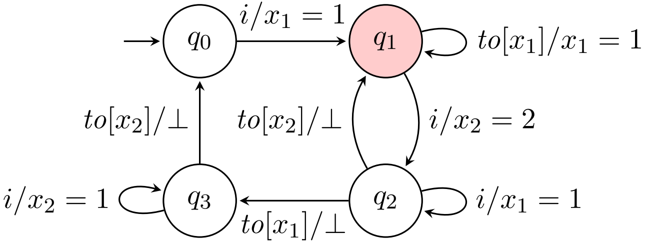
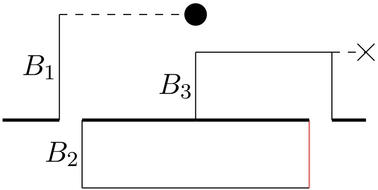
&\xrightarrow{\tau} (q_2, 0 < x_1 = x_2 < 1, \emptyset) \xrightarrow{\tau} (q_2, x_1 = 0 = x_2, \emptyset)
\end{aligned}
$$





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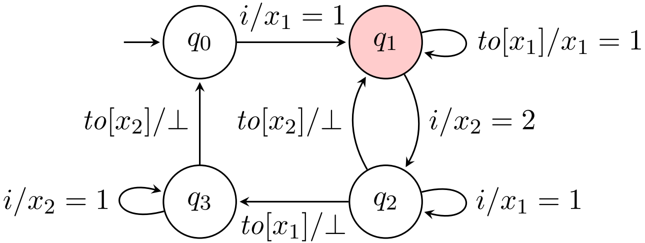
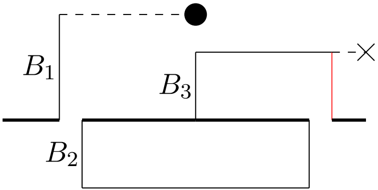
&\xrightarrow{\tau} (q_2, 0 < x_1 = x_2 < 1, \emptyset) \xrightarrow{\tau} (q_2, x_1 = 0 = x_2, \emptyset) \\
&\xrightarrow{(to[x_2], \bot)} (q_1, x_1 = 0, \emptyset)
\end{aligned}
$$





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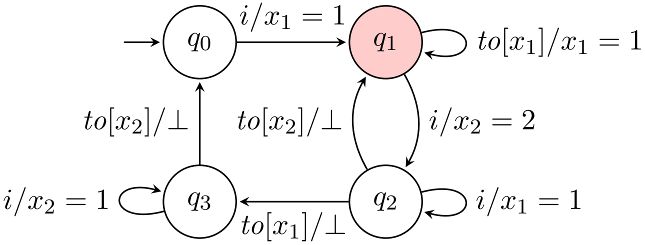
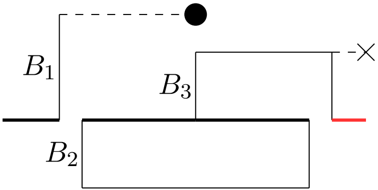
\end{aligned}
$$





$$
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&\xrightarrow{\tau} (q_2, 0 < x_1 < 1 \wedge x_2 - x_1 = 1, \emptyset) \xrightarrow{\tau} (q_2, x_1 = 0 \wedge x_2 = 1, \emptyset) \\
&\xrightarrow{(i,x_1)} (q_2, x_1 = 1 = x_2, \{x_1\}) \xrightarrow{\mathrm{di}[x_1]} (q_2, x_1 = 1 = x_2, \emptyset) \\
&\xrightarrow{\tau} (q_2, 0 < x_1 = x_2 < 1, \emptyset) \xrightarrow{\tau} (q_2, x_1 = 0 = x_2, \emptyset) \\
&\xrightarrow{(to[x_2],\bot)} (q_1, x_1 = 0, \emptyset) \xrightarrow{(to[x_1],x_1)} (q_1, x_1 = 1, \emptyset) \xrightarrow{\tau} (q_1, 0 < x_1 < 1, \emptyset)
\end{aligned}
$$

We need to express the following:

- Two symbols are in concurrency iff there is no τ in between.
- Two symbols are in the same block iff there is no transition using the timer of the block.
- There exists a cycle in the block graph.

The formula can be written with three quantifiers alternations $\rightsquigarrow$ 3EXP.

Theorem 5 (Contribution)

Fix an automaton and a state q. Deciding whether there exists an execution of the automaton that reaches q is PSPACE*-complete.*

Theorem 6 (Contribution)

Deciding whether an AT contains an execution in which some concurrency cannot be avoided is PSPACE*-hard and in* 3EXP*.*

Thank you!

For all details, see Bruyère et al., "Automata with Timers", 2023.

References I

Angluin, Dana. "Learning Regular Sets from Queries and Counterexamples". In: *Inf. Comput.* 75.2 (1987), pp. 87–106. DOI: 10.1016/0890-5401(87)90052-6.

Bruyère, Véronique et al. "Automata with Timers". In: *CoRR* abs/2305.07451 (2023). DOI: 10.48550/arXiv.2305.07451. arXiv: 2305.07451. URL: https://doi.org/10.48550/arXiv.2305.07451.

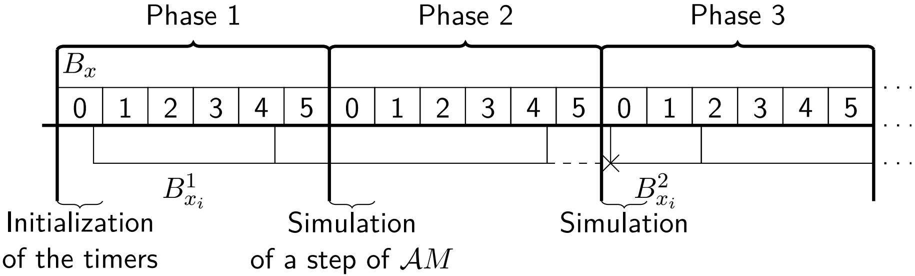

Figure 15: The beginning of a run for the reachability PSPACE-hardness proof.

Let $\mathcal{A} = (X, I, Q, q_0, \chi, \delta)$ be an automaton with timers. For a timer $x \in X$, c_x denotes the largest constant to which x is updated in $\mathcal{A}$. Let $C = \max_{x \in X} c_x$.
Two valuations κ and κ' are said *timer-equivalent*, noted $\kappa \cong \kappa'$, iff $\mathsf{dom}(\kappa) = \mathsf{dom}(\kappa')$ and the following hold for all $x_1, x_2 \in \mathsf{dom}(\kappa)$:

- $\lfloor \kappa(x_1) \rfloor = \lfloor \kappa'(x_1) \rfloor$,
- $\mathrm{frac}(\kappa(x_1)) = 0$ iff $\mathrm{frac}(\kappa'(x_1)) = 0$,
- $\mathrm{frac}(\kappa(x_1)) \leq \mathrm{frac}(\kappa(x_2))$ iff $\mathrm{frac}(\kappa'(x_1)) \leq \mathrm{frac}(\kappa'(x_2))$.

A *timer region* for $\mathcal{A}$ is an equivalence class of timer valuations induced by $\cong$. We lift the relation to configurations: $(q, \kappa) \cong (q', \kappa')$ iff $\kappa \cong \kappa'$ and $q = q'$. Finally, $[\![(q, \kappa)]\!]_{\cong}$ denotes the equivalence class of (q, κ).
We are now able to define a finite automaton called the *region automaton* of $\mathcal{A}$ and denoted $\mathcal{R}$. The alphabet of $\mathcal{R}$ is $\Sigma = \{\tau\} \cup \hat{I}$ where τ is a special symbol used in non-zero delay transitions. Formally, $\mathcal{R}$ is the finite automaton (Σ, S, s_0, Δ) where:

- $S = \{(q, \kappa) \mid q \in Q, \kappa \in \mathsf{Val}(\chi(q))\}_{/\cong}$, i.e., the quotient of the configurations by $\cong$, is the set of states,
- $s_0 = (q_0, [\![\kappa_0]\!]_{\cong})$ with κ_0 the empty valuation, is the initial state,
- the set of transitions $\Delta \subseteq S \times \Sigma \times S$ includes $([\![(q, \kappa)]\!]_{\cong}, \tau, [\![(q, \kappa')]\!]_{\cong})$ if $(q, \kappa) \xrightarrow{d} (q, \kappa')$ in $\mathcal{A}$ whenever $d > 0$, and $([\![(q, \kappa)]\!]_{\cong}, i, [\![(q', \kappa')]\!]_{\cong})$ if $(q, \kappa) \xrightarrow[u]{i} (q', \kappa')$ in $\mathcal{A}$.

Lemma 7

Let $\mathcal{A} = (X, I, Q, q_0, \chi, \delta)$ *be an automaton with timers and* $\mathcal{R}$ *be its region automaton.*

1. *The size of* $\mathcal{R}$ *is linear in* $|Q|$ *and exponential in* $|X|$*. That is,* $|S|$ *is smaller than or equal to* $|Q| \cdot |X|! \cdot 2^{|X|} \cdot (C+1)^{|X|}$*.*
2. *There is a timed run* ρ *of* $\mathcal{A}$ *that begins in* (q, κ) *and ends in* (q', κ') *iff there is a run* ρ' *of* $\mathcal{R}$ *that begins in* $[\![(q, \kappa)]\!]_{\cong}$ *and ends in* $[\![(q', \kappa')]\!]_{\cong}$*.*

Corollary 8

Let $\mathcal{A}$ be an automaton with timers and $\rho \in ptruns(\mathcal{A})$ be a padded timed run with races. Suppose that G_ρ is cyclic. Then there exists a cycle $\mathcal{C}$ in G_ρ such that

- *any block of $\mathcal{C}$ participates in exactly two races described by this cycle,*
- *for any race described by $\mathcal{C}$, exactly two blocks of $\mathcal{C}$ participate in the race,*
- *the blocks $B = (k_1 \dots k_m, \gamma)$ of $\mathcal{C}$ satisfy either $m \geq 2$, or $m = 1$ and $\gamma = \bullet$.*