Verification of computer systems thanks to state machines

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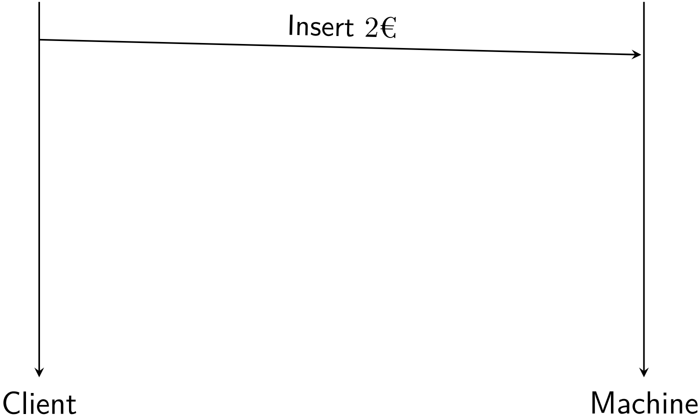
Coffee Machine – Correct execution



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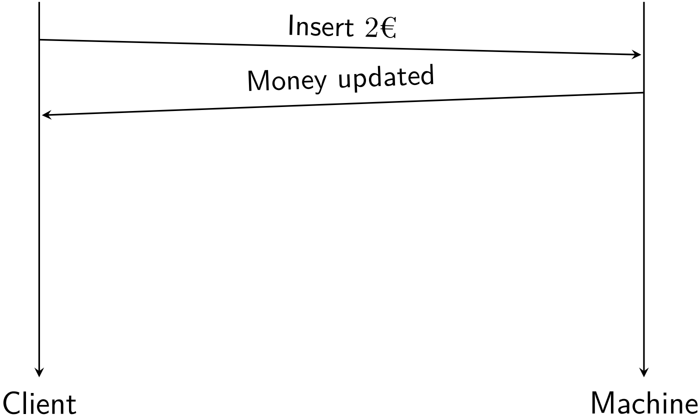

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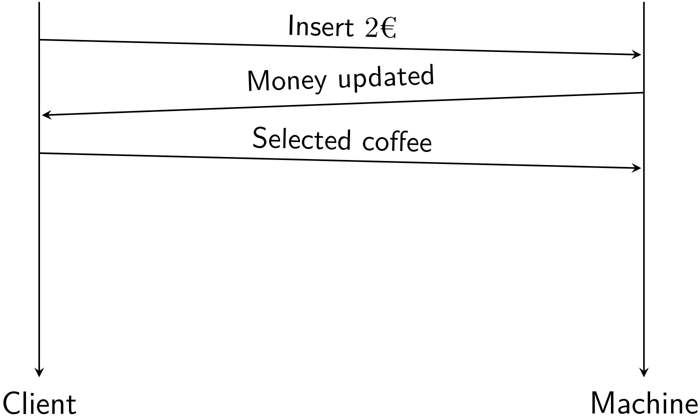

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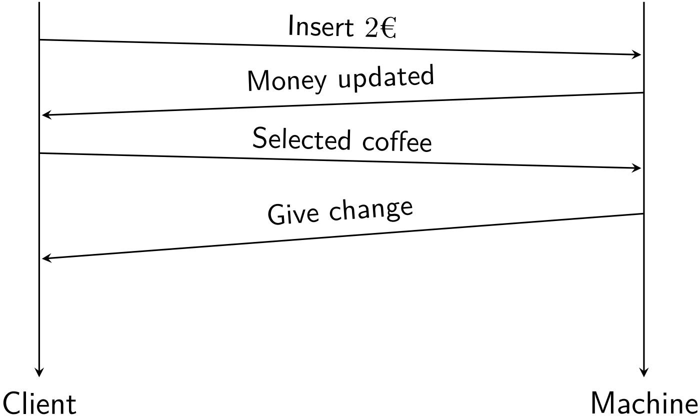

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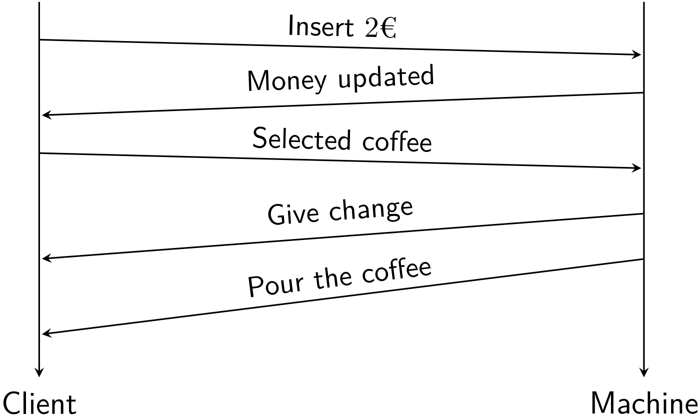

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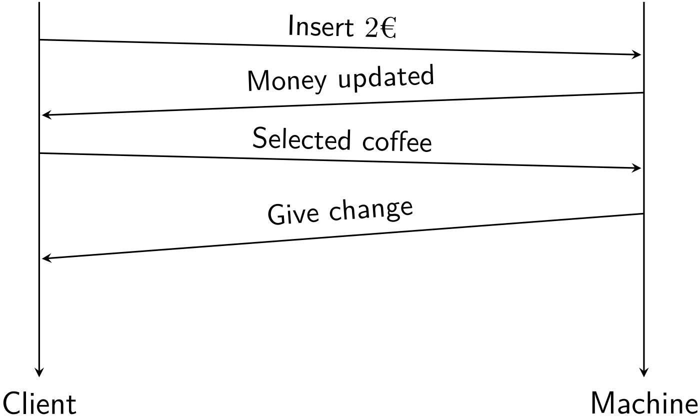

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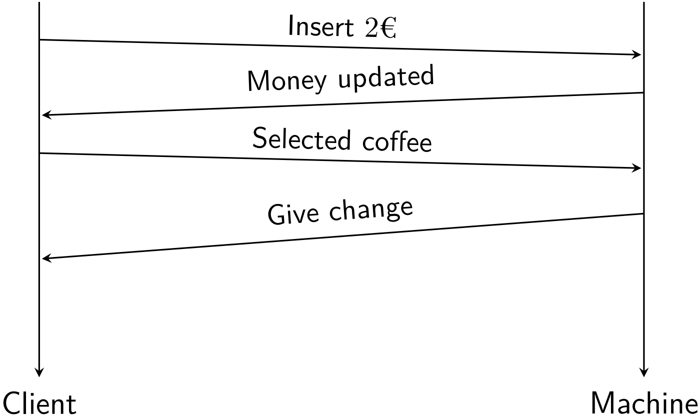

How can we detect the fault as soon as possible?

Detecting faults

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- Construct a model $\mathcal{M}$ of the system.
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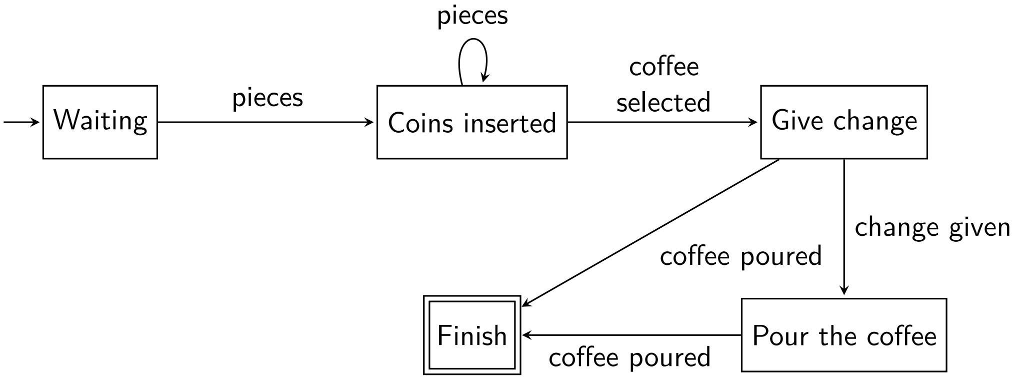
Here, we focus on the construction of the model.

A model for the coffee machine



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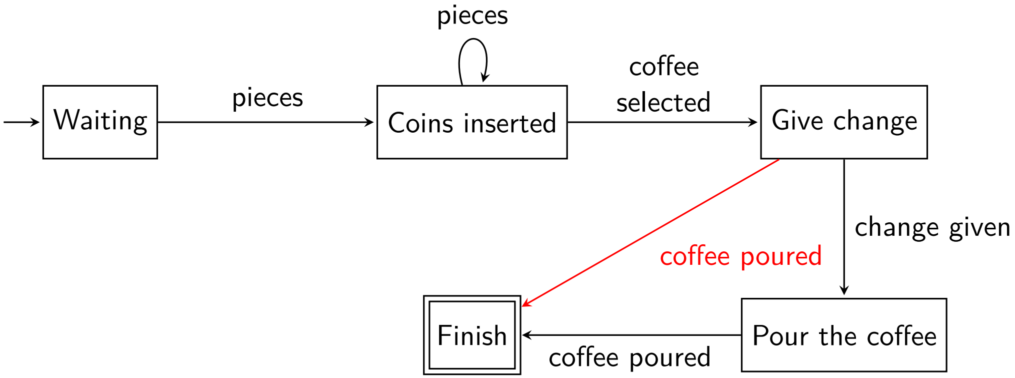

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A language L over an alphabet Σ is a set of words.

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$\Sigma = \{a, b\}$ is an alphabet.
$w = ababb$ is a word over Σ.
$L' = \{\varepsilon, a, b\}$ and $L = \{w \mid w \text{ has an even number of } a \text{ and an odd number of } b\}$ are two languages over Σ.

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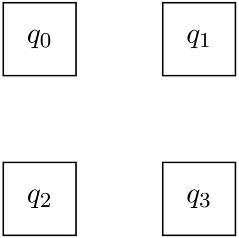

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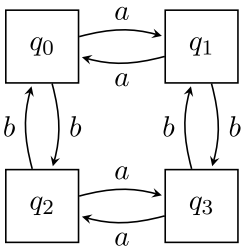

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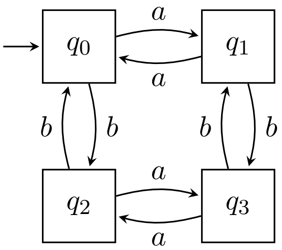

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- $F \subseteq Q$ the set of final states.

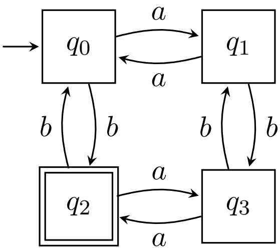

Figure 1: A DFA $\mathcal{A}$.

Let $w = a_1 a_2 \ldots, a_n \in \Sigma^*$. The run of $\mathcal{A}$ over w is the sequence of states

$$p_1 \xrightarrow{a_1} p_2 \xrightarrow{a_2} p_3 \xrightarrow{a_3} \ldots \xrightarrow{a_n} p_{n+1}$$

such that $p_1 = q_0$ and $\forall i, \delta(p_i, a_i) = p_{i+1}$.

Example 2

Let $w = ababb$. The corresponding run is

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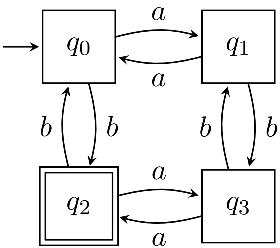

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such that $p_1 = q_0$ and $\forall i, \delta(p_i, a_i) = p_{i+1}$.
If $p_{n+1} \in F$, then w is accepted by $\mathcal{A}$.

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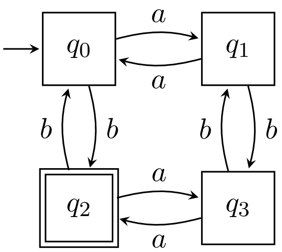

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Which model?

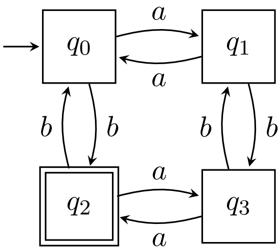
The language of $\mathcal{A}$ is the set of all accepted words, i.e.,

$$\mathcal{L}(\mathcal{A}) = \{w \mid \exists p \in F, q_0 \xrightarrow{w} p\}.$$

Example 3

The language of $\mathcal{A}$ is

$$\mathcal{L}(\mathcal{A}) = \{w \mid w \text{ has an even number of } a \text{ and an odd number of } b\}.$$



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Infinite table

Let $L = \{w \mid w$ has an even number of a and an odd number of $b\}$.

Let $u \in \Sigma^*$. For all $w \in \Sigma^*$, we check whether $uw \in L$.
We construct a table where the rows are the u and the columns the w.

Infinite table

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	ε	a	b	aa	ab	ba	bb	$\dots$
ε	0	0	1	0	0	0	0	$\dots$
a	0	0	0	0	1	1	0	$\dots$
b	1	0	0	1	0	0	1	$\dots$
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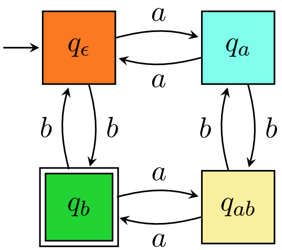

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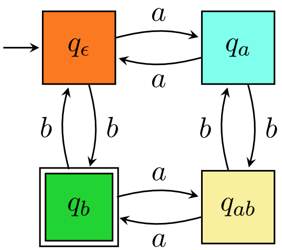

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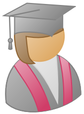
$\hookrightarrow$ A finite table is enough.

How to learn a table?

Learner



Teacher

Knows L

Figure 2: Angluin's framework.[1]

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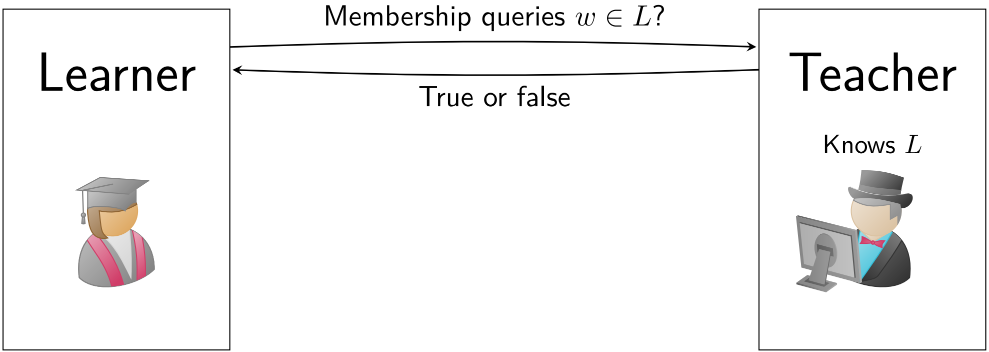

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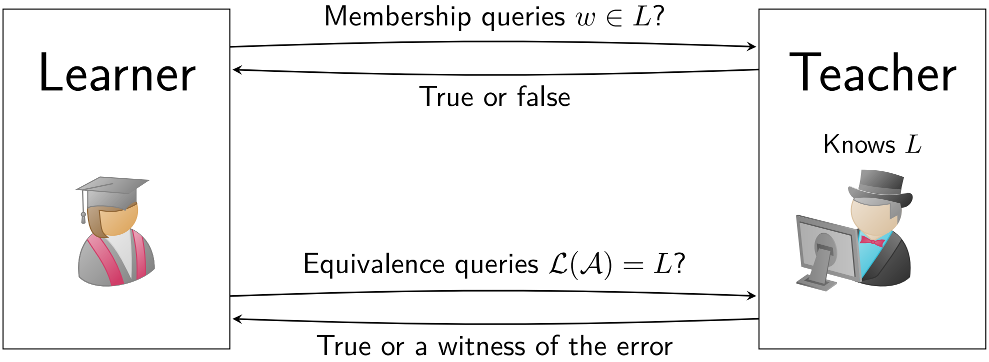

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$\hookrightarrow$ It depends on the exact problem.

JSON Documents

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{
  "title": "Verification by state machines",
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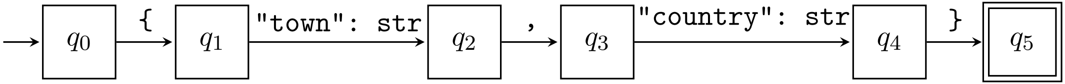

Figure 3: An automaton for the value of "place".

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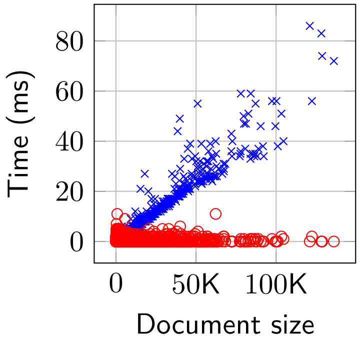

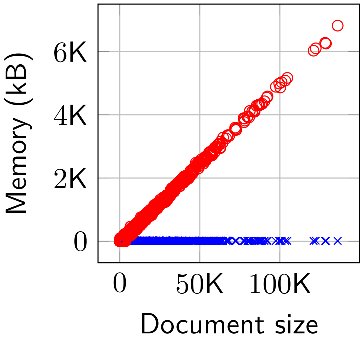

Figure 4: Experimental results for our JSON documents validation algorithm. Blue crosses give the values for our algorithm, and the red circles for the "classical" algorithm.

Thank you!

References I

Angluin, Dana. "Learning Regular Sets from Queries and Counterexamples". In: *Inf. Comput.* 75.2 (1987), pp. 87–106. DOI: 10.1016/0890-5401(87)90052-6. URL: https://doi.org/10.1016/0890-5401(87)90052-6.

Bruyère, Véronique, Guillermo A. Pérez, and Gaëtan Staquet. "Validating Streaming JSON Documents with Learned VPAs". In: *Tools and Algorithms for the Construction and Analysis of Systems*. Ed. by Sriram Sankaranarayanan and Natasha Sharygina. Cham: Springer Nature Switzerland, 2023, pp. 271–289. ISBN: 978-3-031-30823-9.