Automata with Timers

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Many computer systems have timing constraints:

- Network protocols;
- Schedulers;
- Embedded systems;
- In general, real-time systems.

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Well-known model for these systems: timed automata.[1]

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In short: finite automata augmented with clocks that can be reset or used in guards along transitions and states.

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In short: finite automata augmented with clocks that can be reset or used in guards along transitions and states.

BUT timed automata are hard to construct and understand.

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We focus on properties that can be represented with timers: automata with timers.

Timed automata

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Automata with timers

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2

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- Clocks go from 0 to infinity;
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Automata with timers

- Timers go from a value set by the transition to 0;
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▶ Timed automata are more expressive;	▶ Automata with timers are more restrictive;
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▶ Timed automata are more expressive;	▶ Automata with timers are more restrictive;
▶ Learning (à la Angluin[2]) timed automata is challenging;	▶ Future work: learning algorithm;
▶ Well-known model.	▶ This work studies some properties of automata with timers.

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An automaton with timers (AT) is a tuple $\mathcal{A} = (X, I, Q, q_0, \chi, \delta)$ where

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- I is the set of actions,

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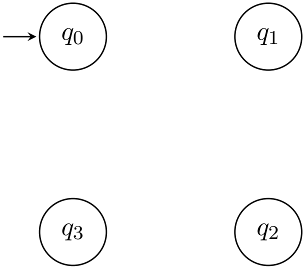

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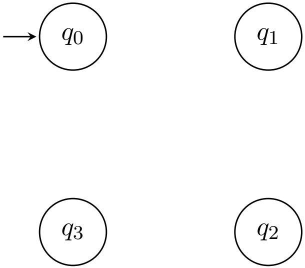

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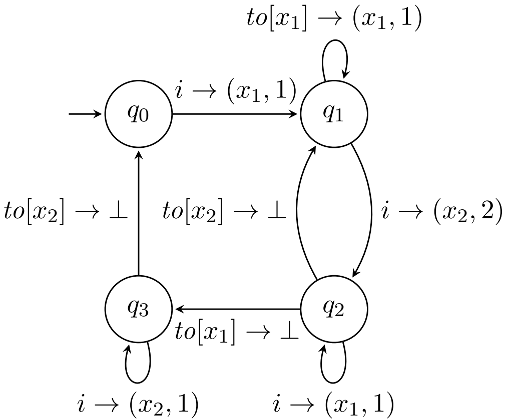
- $\chi : Q \to \mathcal{P}(X)$ gives the active timers of each state,
- δ is the transition function.



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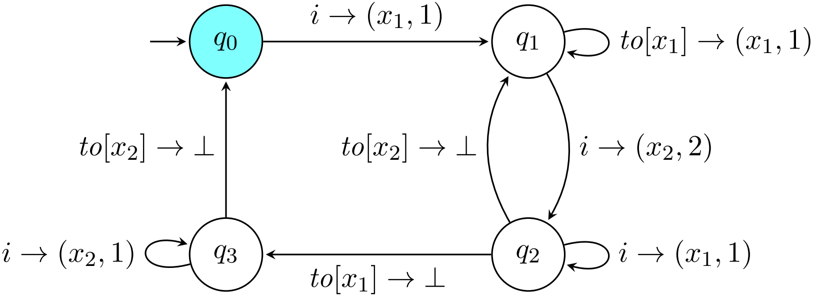
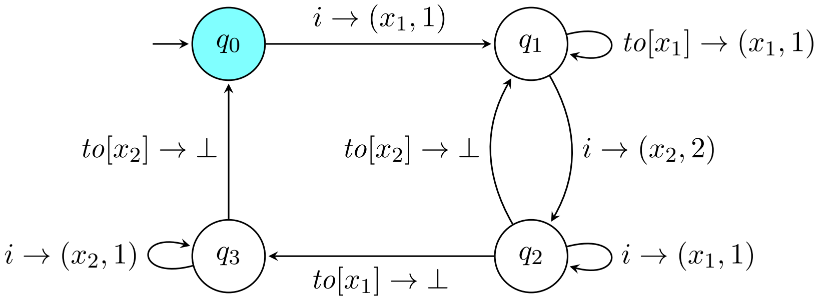


Figure 2: The same AT.

$(q_0, \emptyset)$



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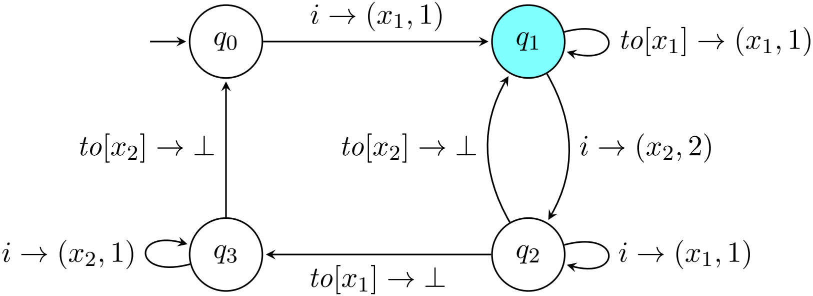
$$(q_0, \emptyset) \xrightarrow{1} (q_0, \emptyset)$$



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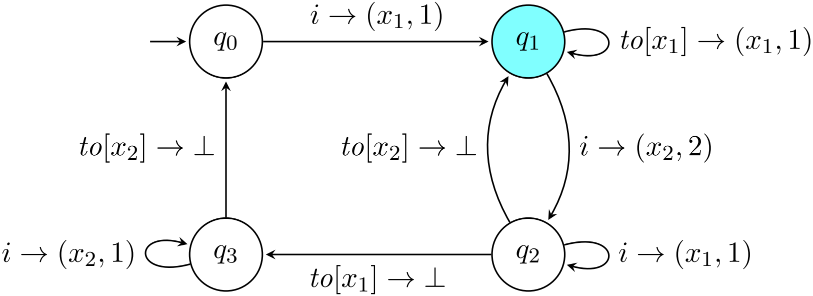
$$(q_0, \emptyset) \xrightarrow{1} (q_0, \emptyset) \xrightarrow[x_1,1]{i} (q_1, x_1 = 1)$$



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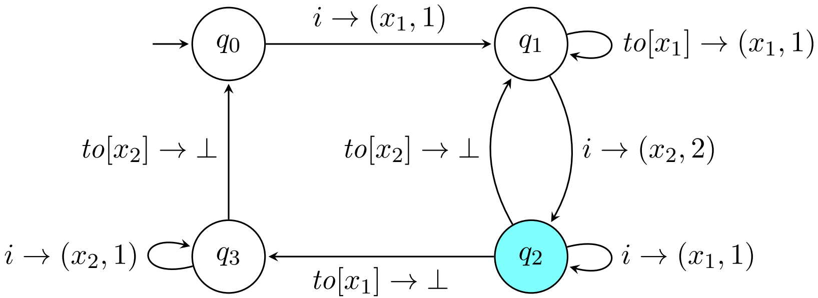

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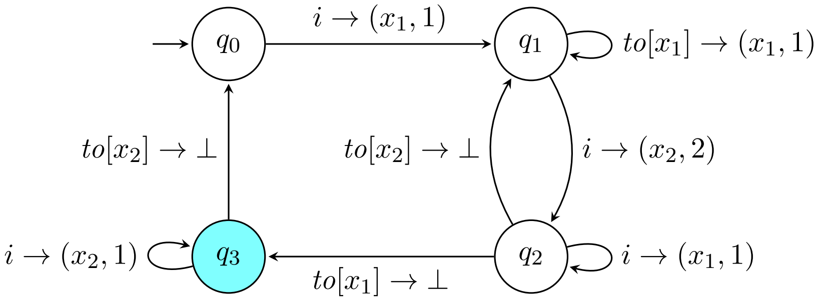

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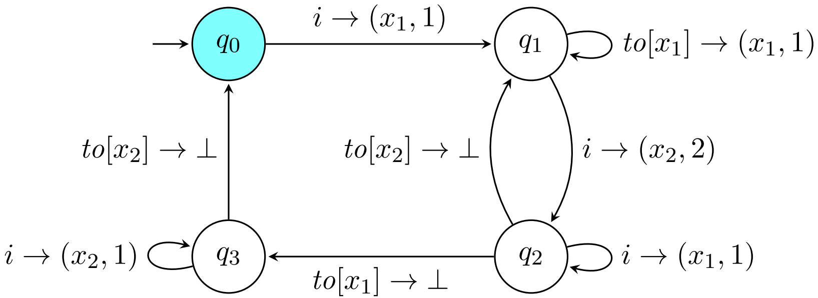

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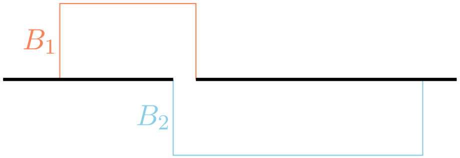

Figure 3: Block representation of the execution.

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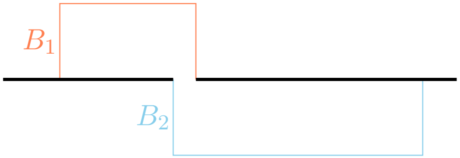

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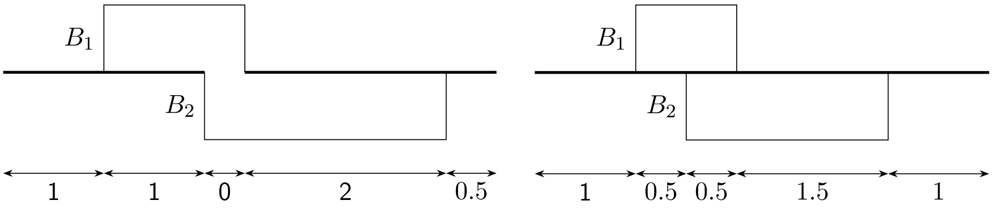
We have concurrent actions.

We can avoid this concurrency and still see the same sequence of actions.



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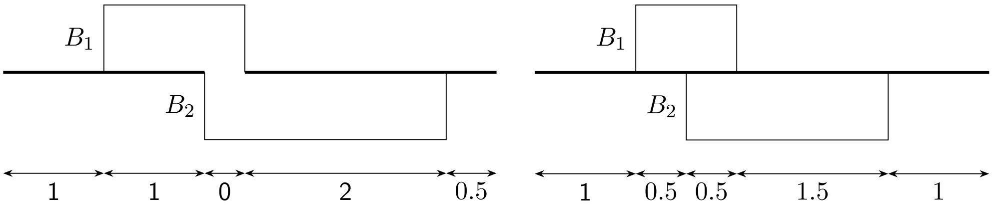

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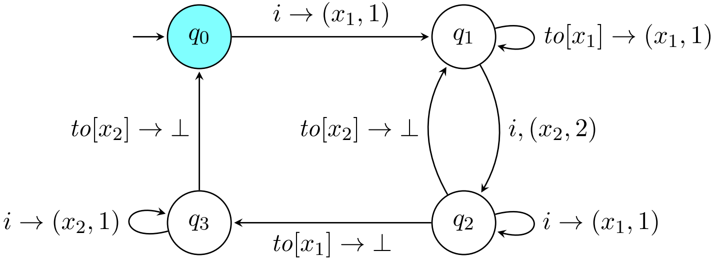
Is it always possible?



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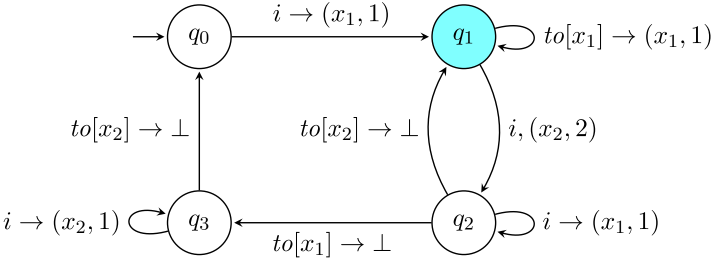

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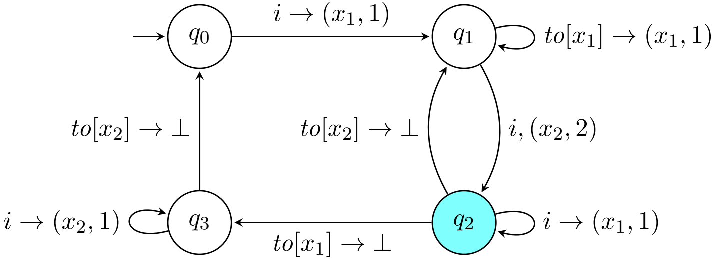

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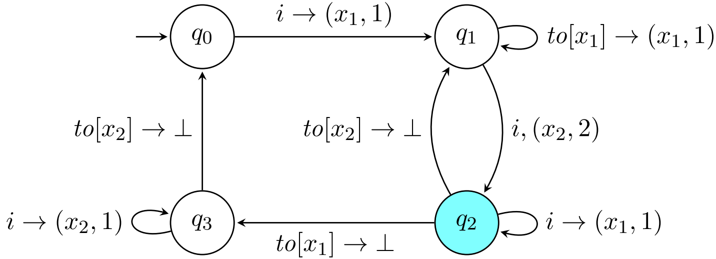

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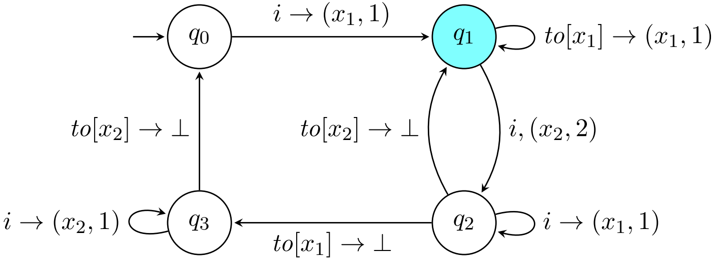

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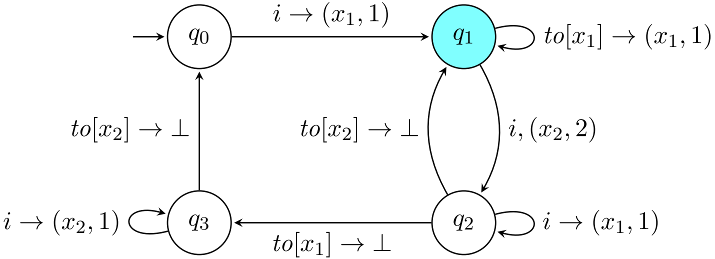

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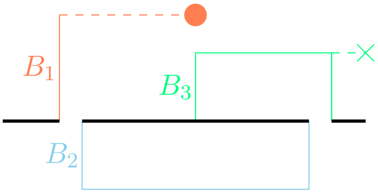

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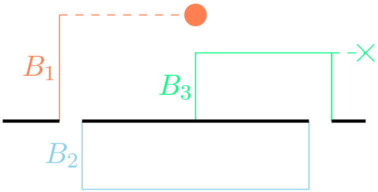

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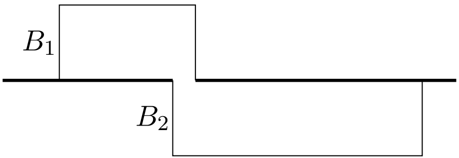
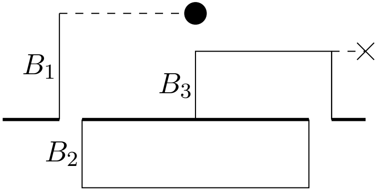
We cannot avoid this concurrency and still see the same sequence of actions.

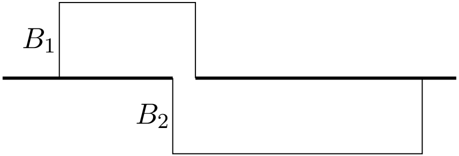


(a) Can be avoided.



(b) Can not be avoided.

Figure 7: Some concurrency can be avoided, some not.



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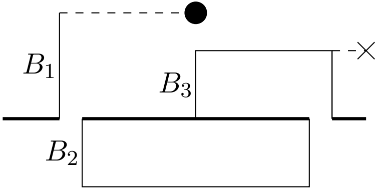

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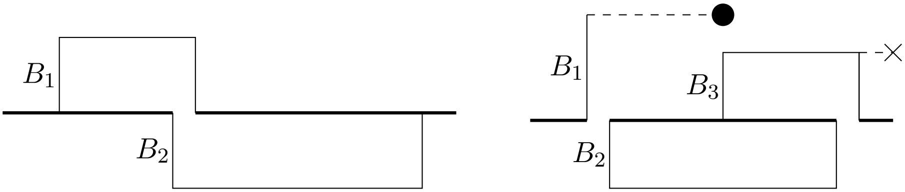
Can we characterize when it is possible to remove the concurrency?



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Yes... But there is not enough time!

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Fix an automaton and a state q. Deciding whether there exists an execution of the automaton that reaches q is PSPACE-*complete.*

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Deciding whether an AT contains an execution in which some concurrency can not be avoided is PSPACE*-hard and in* 3EXP.

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Thank you!

For all details, see Bruyère et al., "Automata with Timers", 2023.

References I

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