Active Learning of Mealy Machines with Timers

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- Embedded systems;
- In general, **real-time** systems.

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BUT timed Mealy machines are hard to construct and understand.

We focus on systems that can be represented with **timers**: **Mealy machines with timers**.

Timed Mealy machines

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Mealy machines with timers

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- We previously studied some properties of Mealy machines with timers;[1]
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[1] Bruyère, Pérez, et al., "Automata with Timers", 2023

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▶ Timed Mealy machines are more expressive;	▶ Mealy machines with timers are more restrictive;
▶ Well-known model;	▶ We previously studied some properties of Mealy machines with timers;[1]
▶ Learning timed Mealy machines is challenging.	▶ This work: learning algorithm.

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A **Mealy machine with timers** (MMT) is a tuple $\mathcal{M} = (X, I, O, Q, q_0, \delta)$ where

- X is the set of **timers**;
- I is the set of **inputs**; the set of all **actions** is:

$$I \cup \{to[x] \mid x \in X\};$$

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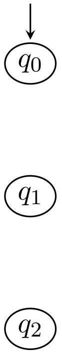
- Q is the finite set of **states**;
- $q_0 \in Q$ is the initial state;



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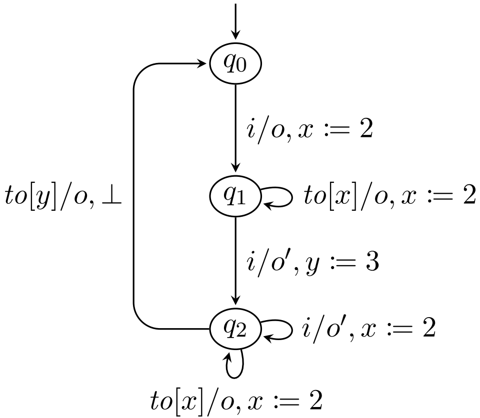
- δ is the transition function.



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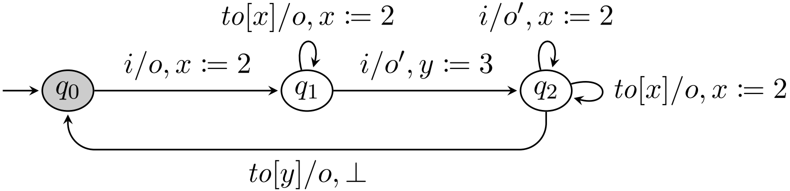
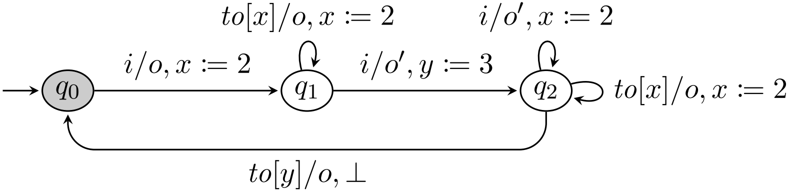
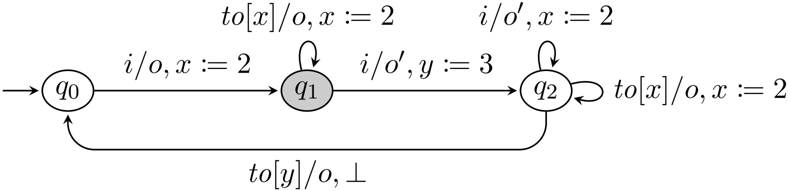


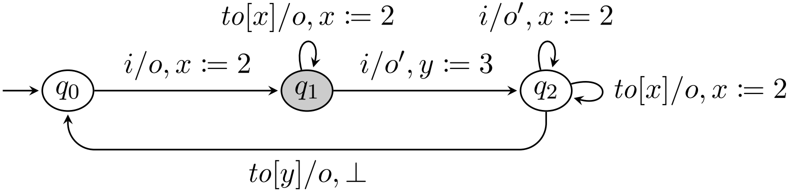
$(q_0, \emptyset)$



$$(q_0, \emptyset) \xrightarrow{1} (q_0, \emptyset)$$



$$(q_0, \emptyset) \xrightarrow{1} (q_0, \emptyset) \xrightarrow{i/o} (q_1, x = 2)$$



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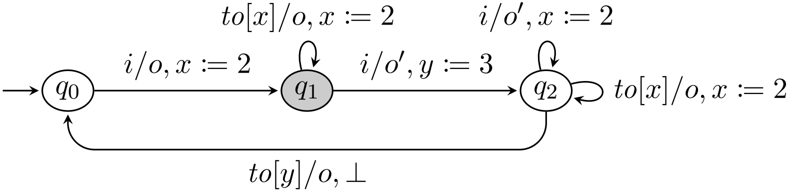

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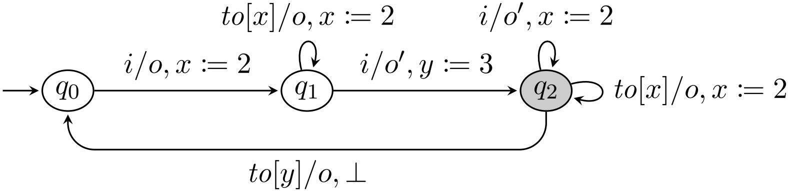

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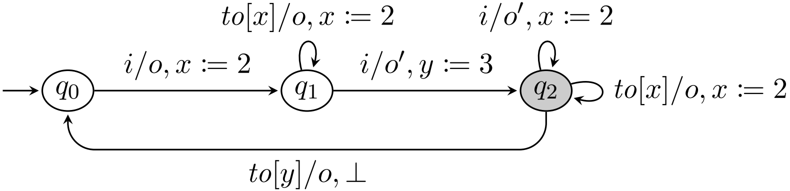

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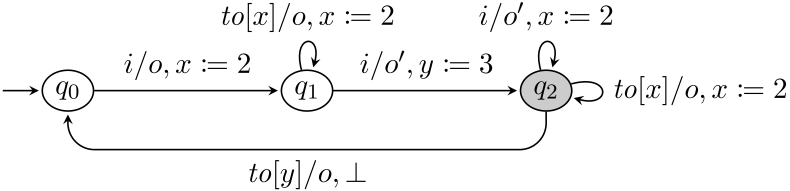

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$$\xrightarrow{i/o'} (q_2, x = 2, y = 1) \xrightarrow{0.5} (q_2, x = 1.5, y = 0.5).$$

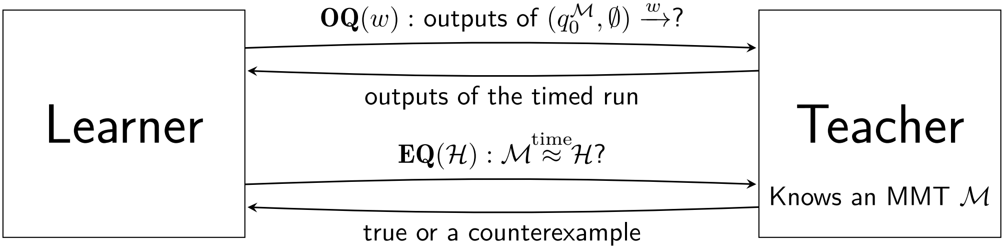

Figure 2: Adaptation of Angluin's framework[2] to MMTs.

[2] Angluin, "Learning Regular Sets from Queries and Counterexamples", 1987; Shahbaz and Groz, "Inferring mealy machines", 2009.

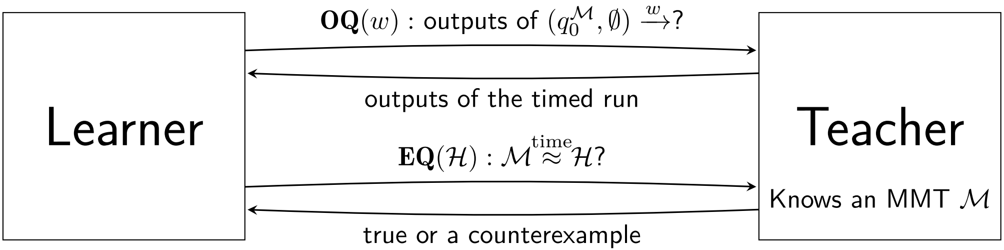

Figure 2: Adaptation of Angluin's framework[2] to MMTs.

Both queries are in the **timed** world... Cumbersome to use!

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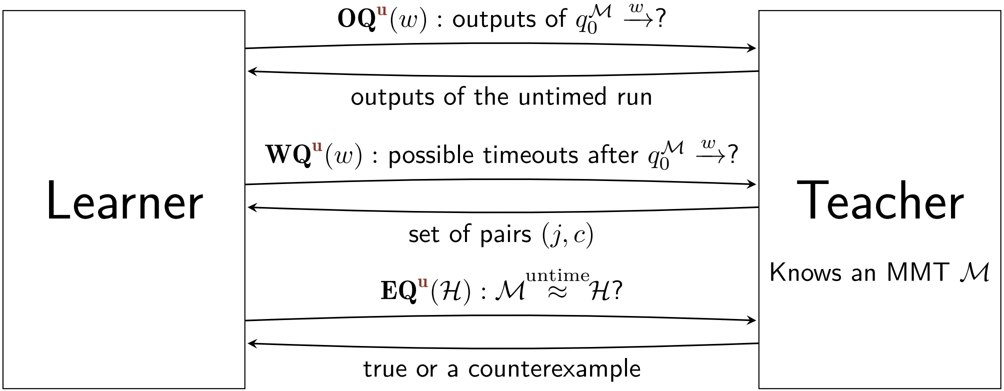

Figure 3: Untimed adaptation of Angluin's framework to MMTs.

We stay in the **untimed** world!

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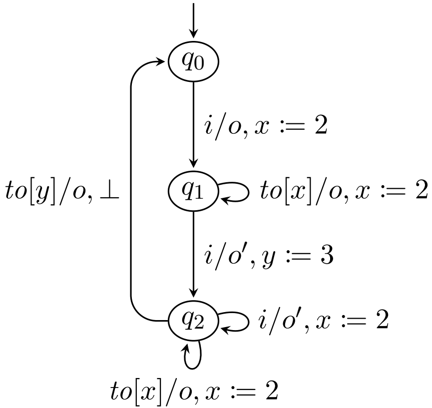
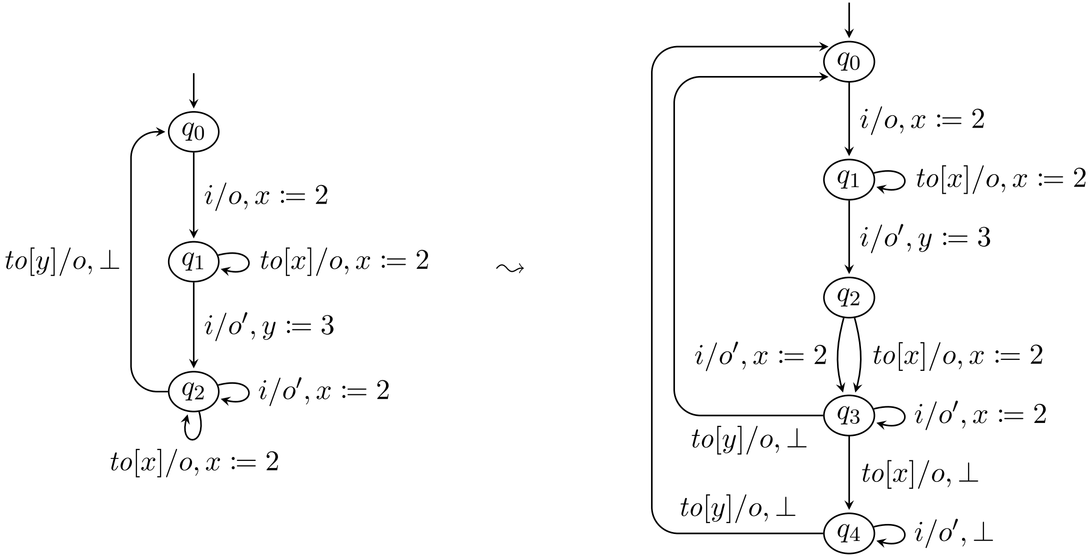
Proposition 2. *It is possible to construct an MMT in which the second condition is satisfied.*


q_0
$i/o, x := 2$
q_1
$to[x]/o, x := 2$
$to[y]/o, \bot$
$i/o', y := 3$
q_2
$i/o', x := 2$
$to[x]/o, x := 2$


q_0
$i/o, x := 2$
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$i/o', y := 3$
q_2
$i/o', x := 2$
$to[x]/o, x := 2$
$to[y]/o, \bot$
$\rightsquigarrow$
q_0
$i/o, x := 2$
q_1
$to[x]/o, x := 2$
$i/o', y := 3$
q_2
$i/o', x := 2$
$to[x]/o, x := 2$
q_3
$i/o', x := 2$
$to[y]/o, \bot$
$to[x]/o, \bot$
$to[y]/o, \bot$
q_4
$i/o', \bot$

We adapt $L^{\#}$ (active learning algorithm for Mealy machines[3]) to MMTs: $L^{\#}_{\mathsf{MMT}}$.

[3] Vaandrager et al., "A New Approach for Active Automata Learning Based on Apartness", 2022.

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Theorem 3. *Let $\mathcal{M}$ be a "good" MMT and ℓ be the length of the longest counterexample returned by the teacher. Then,*

- *the $L^{\#}_{MMT}$ algorithm eventually terminates and returns an MMT $\mathcal{N}$ such that $\mathcal{M} \overset{\text{time}}{\approx} \mathcal{N}$ and whose size is **polynomial** in $|Q^{\mathcal{M}}|$ and **factorial** in $|X^{\mathcal{M}}|$, and*
- *in time and number of untimed queries **polynomial** in $|Q^{\mathcal{M}}|, |I|$, and ℓ, and **factorial** in $|X^{\mathcal{M}}|$.*

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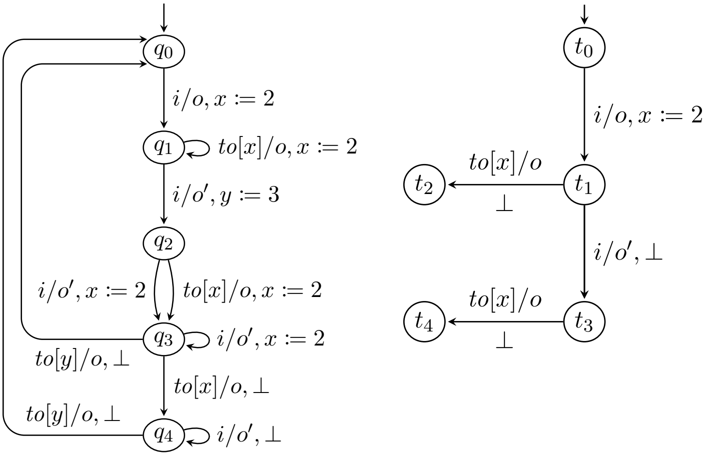
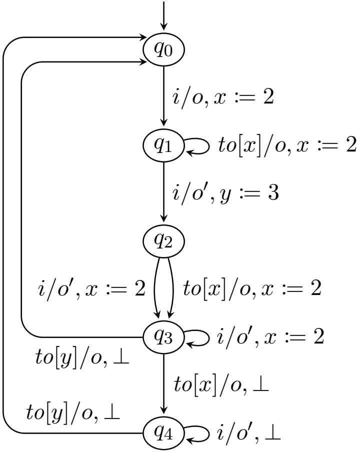
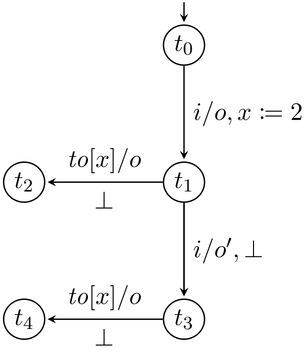


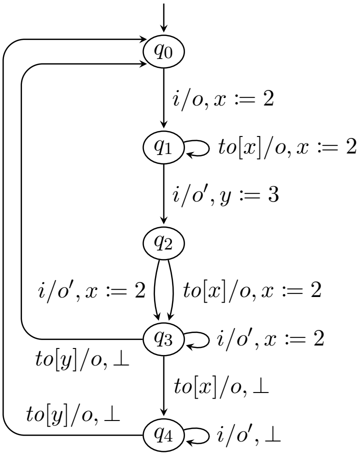
We want to add $i \cdot i \cdot i$ and the potential timeouts.

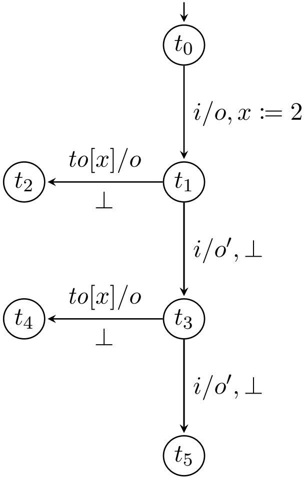




We want to add $i \cdot i \cdot i$ and the potential timeouts.

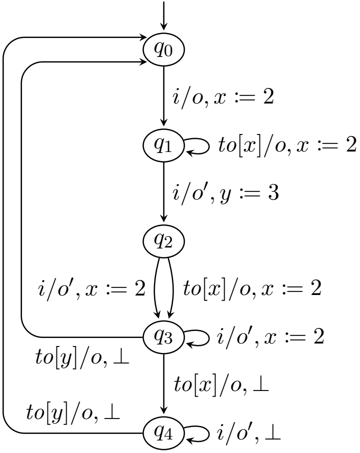
- $\mathbf{OQ}^{\mathbf{u}}(i \cdot i \cdot i) \rightsquigarrow o \cdot o' \cdot o'$.

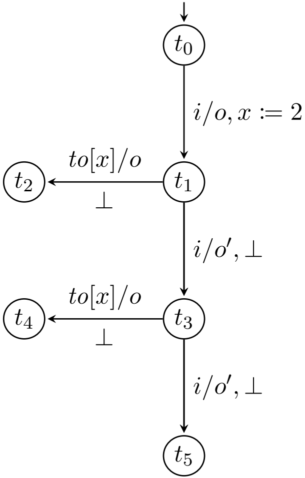




We want to add $i \cdot i \cdot i$ and the potential timeouts.

- $\mathbf{OQ^u}(i{\cdot}i{\cdot}i) \rightsquigarrow o{\cdot}o'{\cdot}o'$.
- So, $t_3 \xrightarrow[\bot]{i/o'} t_5$.





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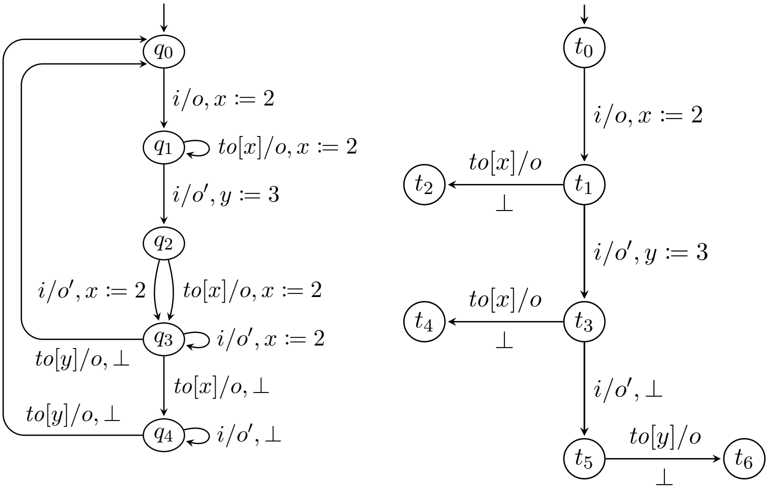

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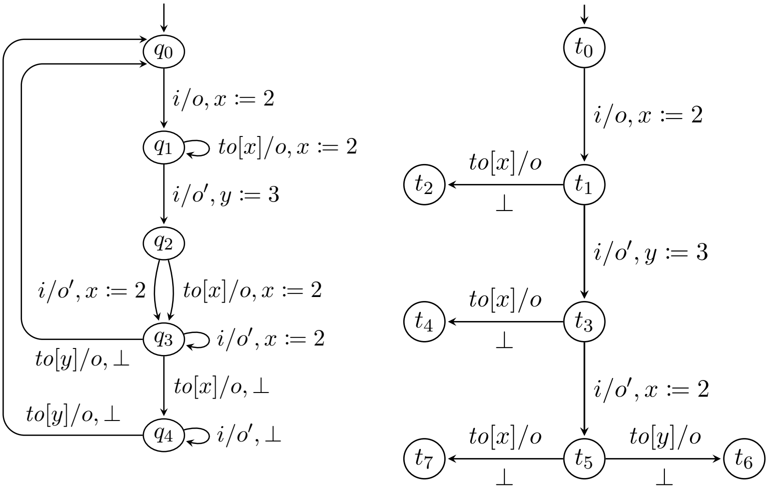
- $\mathbf{WQ^u}(i \cdot i \cdot i) \rightsquigarrow \{(2, 3), (3, 2)\}$.
- So, $t_1 \xrightarrow{i} t_3$ starts a timer at constant 3.



We want to add $i \cdot i \cdot i$ and the potential timeouts.

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- So, $t_1 \xrightarrow{i} t_3$ starts a timer at constant 3.
- And $t_3 \xrightarrow{i} t_5$ starts a timer at constant 2.

We implemented $L^{\#}_{\mathsf{MMT}}$ in Rust[4] and ran some experiments.

Model	$\lvert Q\rvert$	$\lvert I\rvert$	$\lvert X\rvert$	$\lvert\mathbf{WQ^u}\rvert$	$\lvert\mathbf{OQ^u}\rvert$	$\lvert\mathbf{EQ^u}\rvert$	Time[msecs]
AKM	4	5	1	22	35	2	684
CAS	8	4	1	60	89	3	1344
Light	4	2	1	10	13	2	302
PC	8	9	1	75	183	4	2696
TCP	11	8	1	123	366	8	3182
Train	6	3	1	32	28	3	1559
Running example	3	1	2	11	5	2	1039
FDDI 1-station	9	2	2	32	20	1	1105
Oven	12	5	1	907	317	3	9452
WSN	9	4	1	175	108	4	3291

[4] `https://gitlab.science.ru.nl/bharat/mmt_lsharp`.

Still work to be done:

- Further experiments with more timers,
- Simplify the learning algorithm as much as possible.

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Thank you!

For all details, see

Bruyère, Garhewal, et al., "Active Learning of Mealy Machines with Timers", 2024.

Part I – Appendix

Appendix

References I

- Angluin, Dana. "Learning Regular Sets from Queries and Counterexamples". In: *Inf. Comput.* 75.2 (1987), pp. 87–106. DOI: 10.1016/0890-5401(87)90052-6.
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Vaandrager, Frits W. et al. "A New Approach for Active Automata Learning Based on Apartness". In: *Tools and Algorithms for the Construction and Analysis of Systems - 28th International Conference, TACAS 2022, Held as Part of the European Joint Conferences on Theory and Practice of Software, ETAPS 2022, Munich, Germany, April 2-7, 2022, Proceedings, Part I*. Ed. by Dana Fisman and Grigore Rosu. Vol. 13243. Lecture Notes in Computer Science. Springer, 2022, pp. 223–243. DOI: 10.1007/978-3-030-99524-9_12.