Active Learning of Mealy Machines with Timers

Véronique Bruyère, Bharat Garhewal, Guillermo A. Pérez,
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February 6, 2025

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In short: finite Mealy machines augmented with **clocks** that can be reset or used in guards along transitions and states.

BUT timed Mealy machines are hard to construct and understand.

We focus on systems that can be represented with **timers**: **Mealy machines with timers**.

Timed Mealy machines

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Mealy machines with timers

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Mealy machines with timers

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▶ Timed Mealy machines are more expressive;	▶ Mealy machines with timers are more restrictive;
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- Learning timed Mealy machines is challenging.

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- We previously studied some properties of Mealy machines with timers;[1]
- This work: learning algorithm.

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A **Mealy machine with timers** (MMT) is a tuple $\mathcal{M} = (X, I, O, Q, q_0, \delta)$ where

- X is the set of **timers**;
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$$I \cup \{to[x] \mid x \in X\};$$

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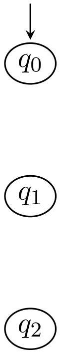

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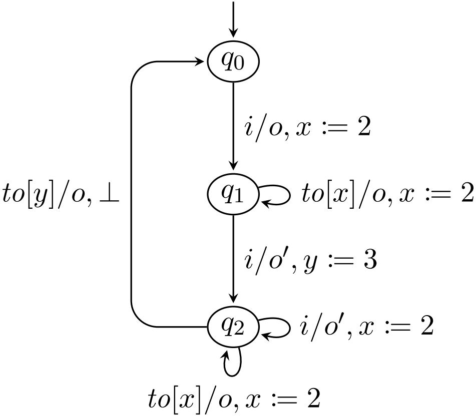
- δ is the transition function.



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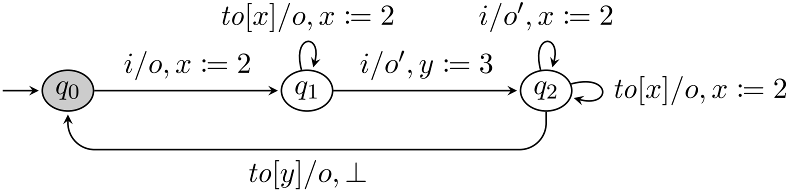
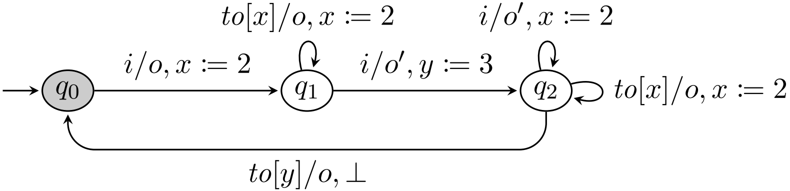
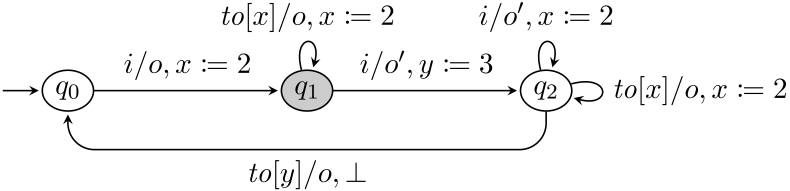


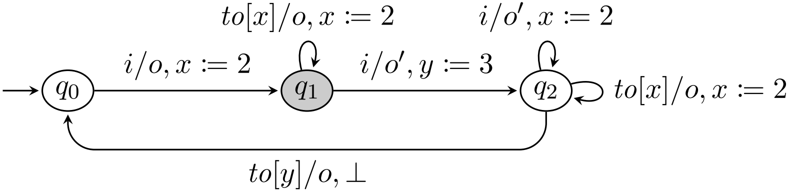
$(q_0, \emptyset)$



$$(q_0, \emptyset) \xrightarrow{1} (q_0, \emptyset)$$



$$(q_0, \emptyset) \xrightarrow{1} (q_0, \emptyset) \xrightarrow{i/o} (q_1, x = 2)$$



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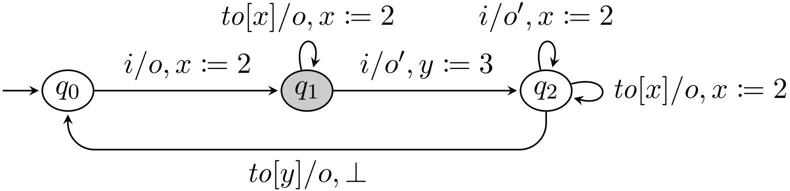

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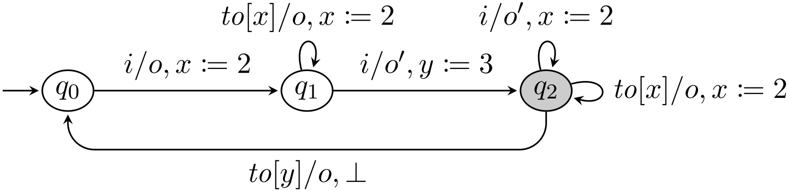

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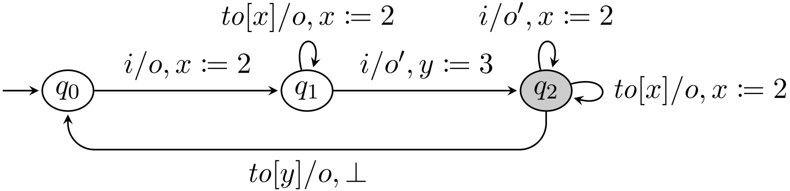

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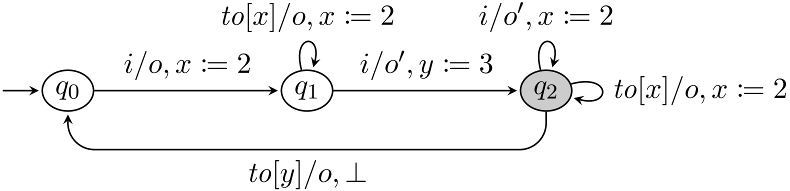

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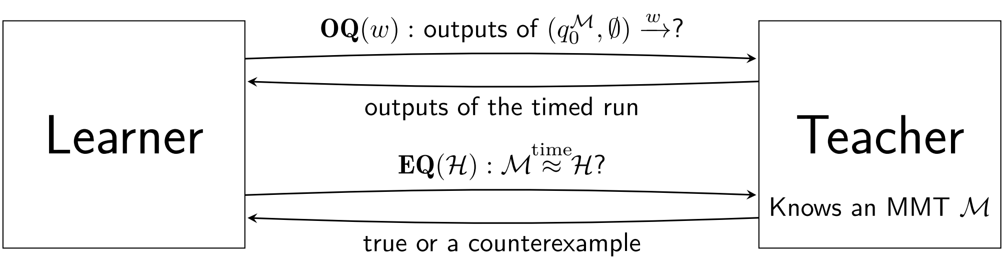
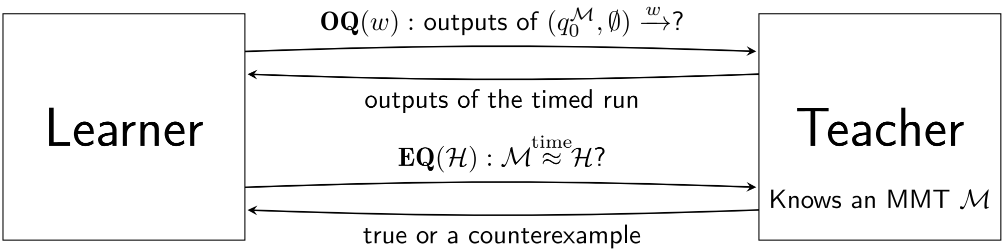


Figure 2: Adaptation of Angluin's framework[2] to MMTs.

[2] Angluin, "Learning Regular Sets from Queries and Counterexamples", 1987.



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Two problems:

- Set of timers of $\mathcal{M}$ is unknown to the learner. $\rightsquigarrow$ Hide the timeouts via the delays.

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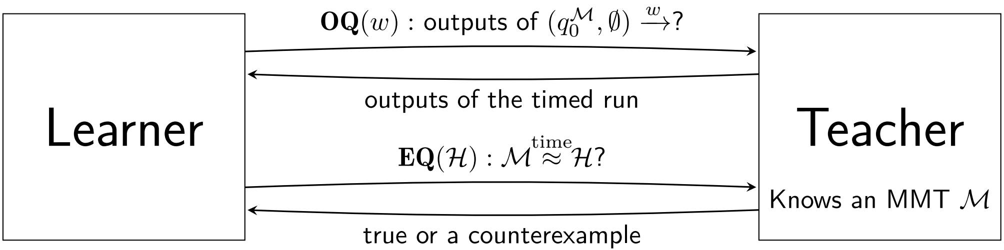

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Two problems:

- Set of timers of $\mathcal{M}$ is unknown to the learner. $\leadsto$ Hide the timeouts via the delays.
- Both queries are in the **timed** world... Cumbersome to use!

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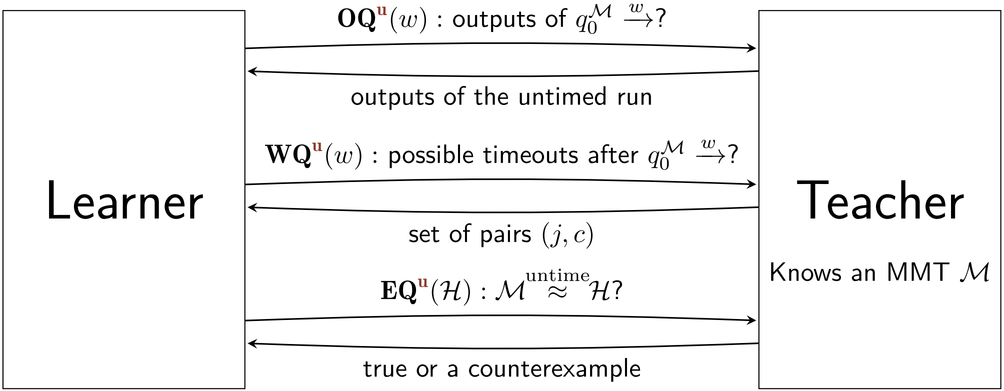

Figure 3: Untimed adaptation of Angluin's framework to MMTs.

We stay in the **untimed** world!

Question. How to hide the names of the timers for the untimed queries?

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$$\cdot \xrightarrow[x:=2]{i/o} \cdot \xrightarrow[x:=2]{to[x]/o} \cdot \xrightarrow[y:=3]{i/o'} \cdot \xrightarrow[x:=2]{to[x]/o'} \cdot \xrightarrow[\bot]{to[y]/o} \cdot$$

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becomes

$$\cdot \xrightarrow[x:=2]{i/o} \cdot \xrightarrow[x:=2]{to[1]/o} \cdot \xrightarrow[y:=3]{i/o'} \cdot \xrightarrow[x:=2]{to[2]/o'} \cdot \xrightarrow[\bot]{to[3]/o} \cdot$$

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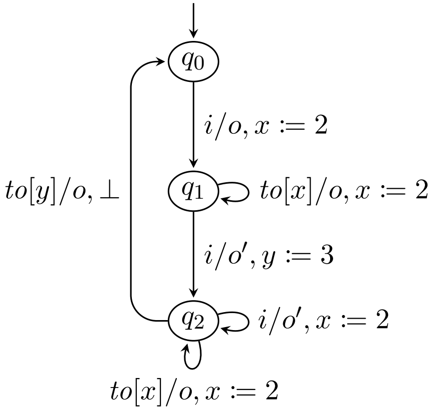
Proposition 2. *It is possible to construct an MMT in which the second condition is satisfied.*


q_0
$i/o, x := 2$
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$to[y]/o, \bot$
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q_2
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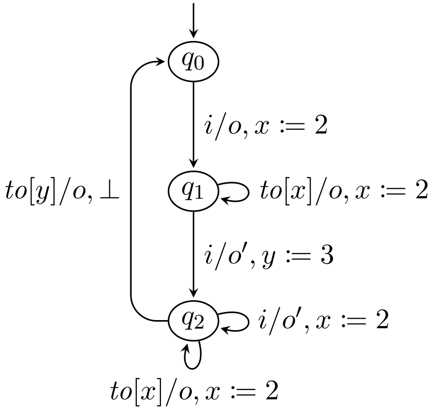
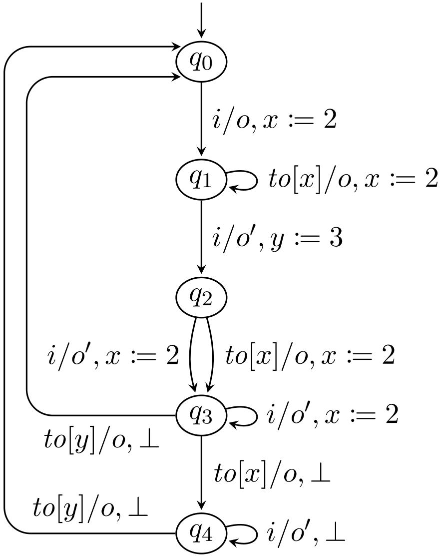


$\rightsquigarrow$



We adapt $L^{\#}$ (active learning algorithm for Mealy machines[3]) to MMTs: $L^{\#}_{\mathsf{MMT}}$.

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Theorem 3. *Let $\mathcal{M}$ be a "good" MMT and ℓ be the length of the longest counterexample returned by the teacher. Then,*

- *the $L^{\#}_{\text{MMT}}$ algorithm eventually terminates and returns an MMT $\mathcal{N}$ such that $\mathcal{M} \overset{\text{time}}{\approx} \mathcal{N}$ and whose size is* ***polynomial*** *in $|Q^{\mathcal{M}}|$ and* ***factorial*** *in $|X^{\mathcal{M}}|$, and*
- *in time and number of untimed queries* ***polynomial*** *in $|Q^{\mathcal{M}}|, |I|$, and ℓ, and* ***factorial*** *in $|X^{\mathcal{M}}|$.*

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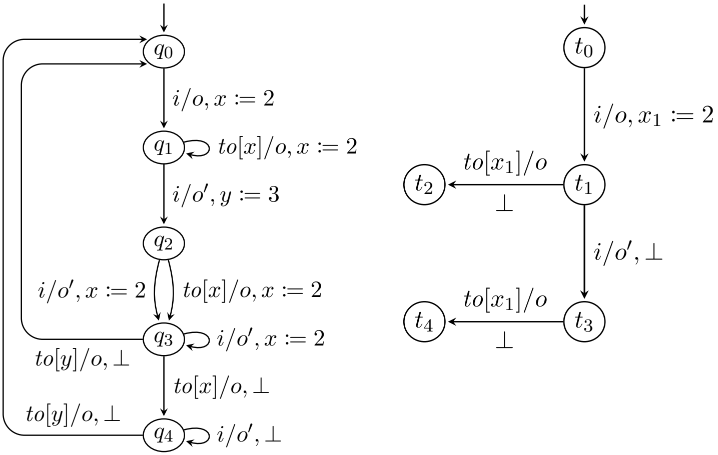
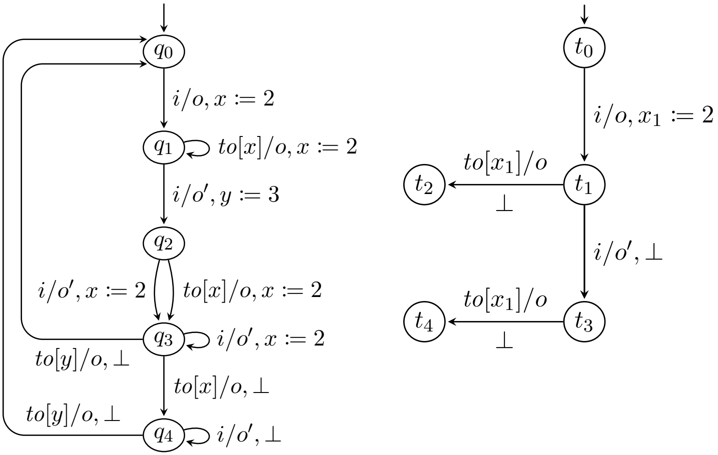


We want to add $i \cdot i \cdot i$ and the potential timeouts.



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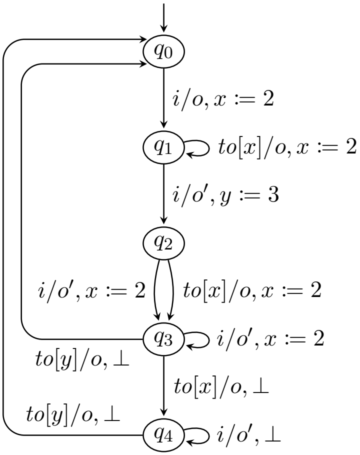
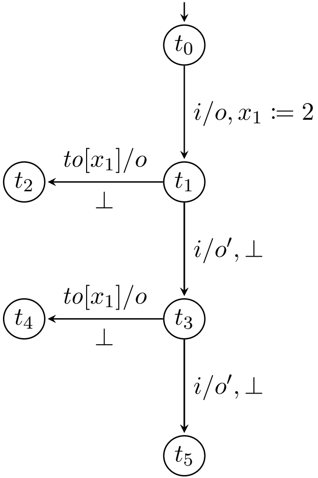
- $\mathbf{OQ^u}(i{\cdot}i{\cdot}i) \rightsquigarrow o{\cdot}o'{\cdot}o'$.





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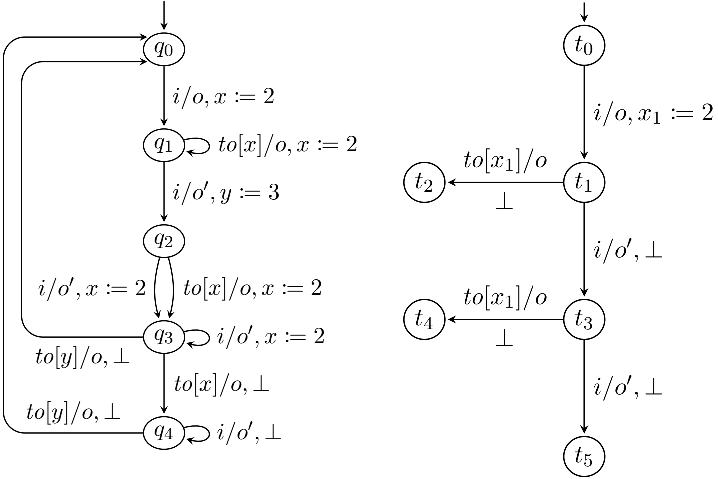

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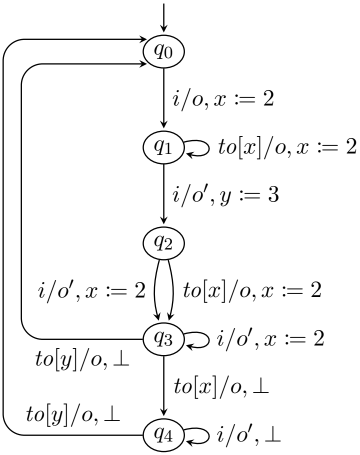
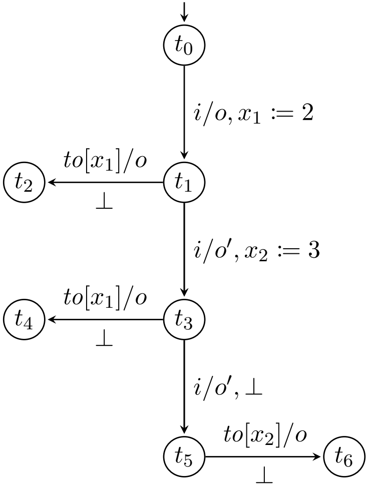
- $\mathbf{OQ^u}(i \cdot i \cdot i) \rightsquigarrow o \cdot o' \cdot o'$.
- So, $t_3 \xrightarrow[\bot]{i/o'} t_5$.
- $\mathbf{WQ^u}(i \cdot i \cdot i)$ $\rightsquigarrow \{(2,3),(3,2)\}$.





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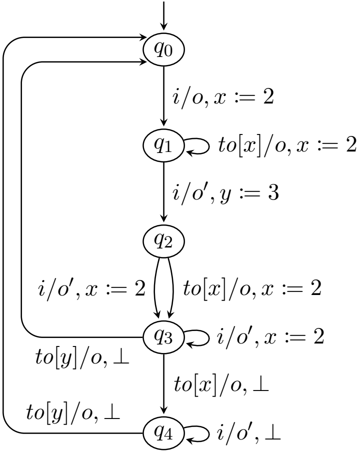
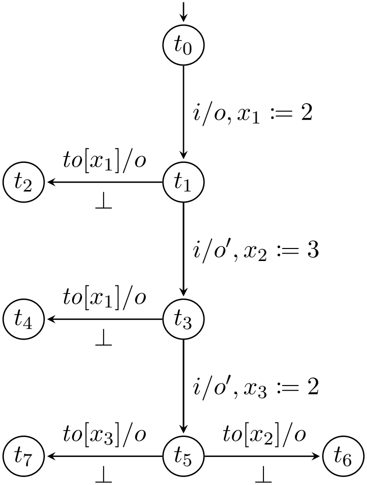
- So, $t_1 \xrightarrow{i} t_3$ starts a timer at constant 3.





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- So, $t_1 \xrightarrow{i} t_3$ starts a timer at constant 3.
- And $t_3 \xrightarrow{i} t_5$ starts a timer at constant 2.

We implemented $L^{\#}_{\mathsf{MMT}}$ in Rust[4] and ran some experiments.

Model	$\|Q\|$	$\|I\|$	$\|X\|$	$\|\mathbf{WQ^u}\|$	$\|\mathbf{OQ^u}\|$	$\|\mathbf{EQ^u}\|$	Time[msecs]
AKM	4	5	1	22	35	2	684
CAS	8	4	1	60	89	3	1344
Light	4	2	1	10	13	2	302
PC	8	9	1	75	183	4	2696
TCP	11	8	1	123	366	8	3182
Train	6	3	1	32	28	3	1559
Running example	3	1	2	11	5	2	1039
FDDI 1-station	9	2	2	32	20	1	1105
Oven	12	5	1	907	317	3	9452
WSN	9	4	1	175	108	4	3291

[4] `https://gitlab.science.ru.nl/bharat/mmt_lsharp`.

Still work to be done:

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- Simplify the learning algorithm as much as possible.

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Thank you!

For all details, see

Bruyère, Garhewal, et al., "Active Learning of Mealy Machines with Timers", 2024.

Part I – Appendix

Appendix

References I

- Angluin, Dana. "Learning Regular Sets from Queries and Counterexamples". In: *Inf. Comput.* 75.2 (1987), pp. 87–106. DOI: 10.1016/0890-5401(87)90052-6.
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